Simulation of Quantum Transport in Periodic and Disordered Systems with Ultracold Atoms

Laurent Sanchez-Palencia

Center for Theoretical Physics
Ecole Polytechnique, CNRS, Univ. Paris-Saclay
F-91128 Palaiseau, France

Acknowledgements

Quantum Matter group @ CPhT

Cécile Crosnier de Bellaistre,
J. Despres,
Hepeng Yao (SIRTEQ),
Laurent Sanchez-Palencia

Former members

Guilhem Boéris (now in start-up company),
Giuseppe Carleo (now at ETH Zurich), Lorenzo Cevolani (now at Univ Göttingen),
Tung-Lam Dao (now in buisness company), Ben Hambrecht (now in Zurich),
Samuel Lellouch (now at Univ Brussels), Lih-King Lim (now at MPQ Dresden),
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Pierre Lugan (now in Ecole Polytechnique de Lausanne), Christian Trefzger (now Scientific advisor at the University of Brussels)

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Overview

RAPID COMMUNICATIONS

PHYSICAL REVIEW A 93, 011601(R) (2016)

Mott transition for strongly interacting one-dimensional bosons in a shallow periodic potential

G. Boéris,[1] L. Gori,[2] M. D. Hoogerland,[3] A. Kumar,[2] E. Lucioni,[2] L. Tanzi,[2] M. Inguscio,[2,4] T. Giamarchi,[5] C. D'Errico,[2] G. Carleo,[1,*] G. Modugno,[2] and L. Sanchez-Palencia[1]

[1] LSP's Quantum Matter group (Palaiseau, France)
[2-4] Inguscio and Modugno's group (LENS; Florence, Italy) and Hoogerland's group
[5] Giamarchi's group (Univ Geneva, Switzerland)

Quantum and numerical simulation of the pinning (Mott) transition

First accurate determination of the Mott critical parameters

Significant deviation from field-theoretic and RG predictions

Validation of a quantum simulator

Open perspectives

Quantum transport in periodic, quasi-periodic, and disordered systems

Mott Transition in Periodic Potentials

Mott transition within Hubbard models

Emblematic superfluid-to-insulator transition

Well documented

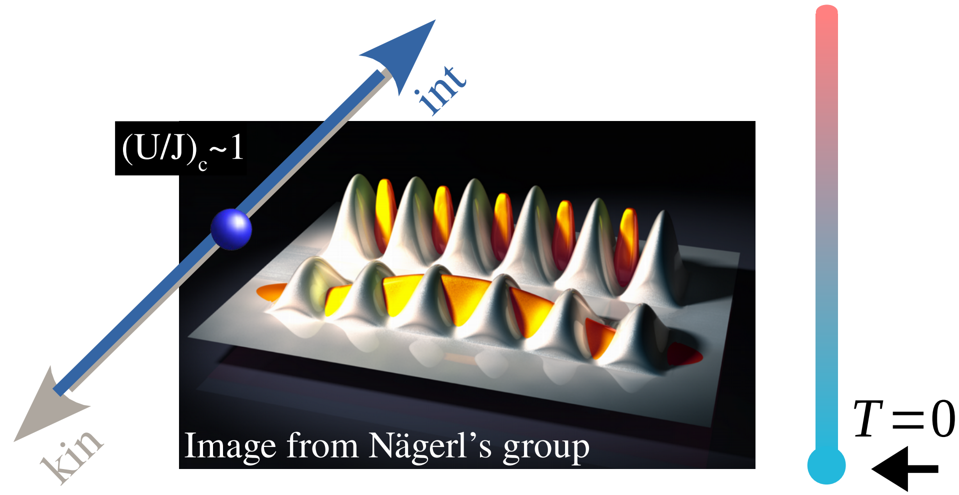
Fisher *et al.*, Phys. Rev. B **40**, 546 (1989)

Spin-1/2 fermions at half filling

Soft bosons at integer filling

Bose-Hubbard model

$$\hat{H} = -\sum_{\langle j,\ell \rangle} J \left(\hat{a}_j^\dagger \hat{a}_\ell + \hat{a}_\ell^\dagger \hat{a}_j \right) + \frac{U}{2} \sum_j \hat{n}_j \left(\hat{n}_j - 1 \right)$$



Transition driven by competition of tunneling and interactions

A unique dimensionless parameter, $U(V_0)/J(V_0)$

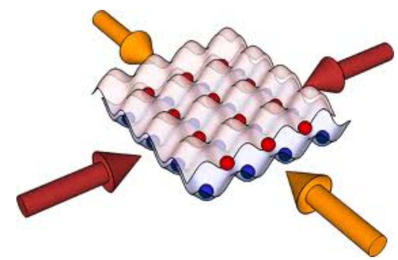
Driven by quantum fluctuations at T=0 (Quantum Phase Transition)

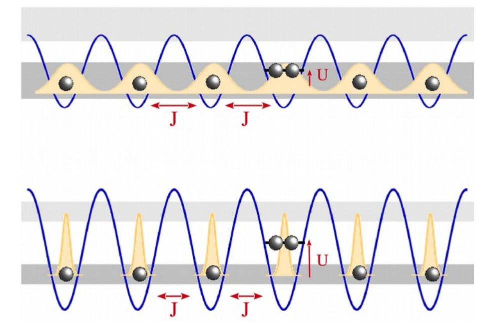
Quantum Simulation of Hubbard Models

Quantum engineering of Hubbard models

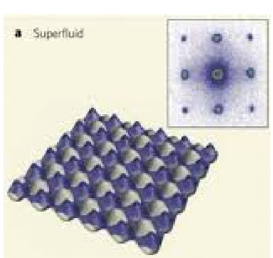
Jaksch *et al.*, Phys. Rev. Lett. **81**, 3108 (1998)

- Ultracold atoms (2-body contact interactions)
- Laser beams create a defectless lattice
- Controllable U/J via V_0
- Accurate measurement (direct imaging, ToF, …)

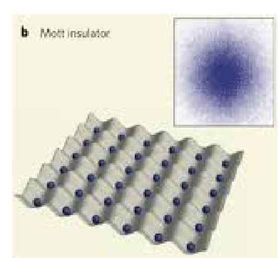




Direct observation of the Mott transition



superfluid



Mott insulator

$(U/J)_c \sim 3.4$ (1D)

U/J

Bosons
Greiner *et al.*, Nature **415**, 39 (2002)
Haller *et al.*, Nature **466**, 597 (2010)

Fermions
Jördens *et al.*, Nature **455**, 204 (2008)
Schneider *et al.*, Science **322**, 1520 (2008)

Strongly-Correlated Systems in One Dimension

Strong correlations have a particular flavor in low dimensions

For reviews, see
Giamarchi, *Quantum Physics in One Dimension* (2004)
Cazalilla *et al.*, Rev. Mod. Phys. **83**,1405 (2011)

Enhancement of quantum interferences due to geometrical reduction of the available space

Enhancement of interactions due to enhancement of the localization energy versus mean-field interaction energy at low density in 1D, $\gamma = mg/\hbar^2 n$

Some important related consequences

Any excitation is collective, no quasi-particle excitations

Diverging susceptibility of homogeneous gases to perturbations that are commensurate with the particle spacing

The superfluid phase becomes unstable against arbitrary weak perturbation

⇨ Mott transition in vanishingly weak periodic potentials, provided interactions are strong enough

A difficult problem

No quasi-exact mean field approach

Strong interactions, continuous space

Renormalization Group Analysis within Tomonaga-Luttinger Liquid Theory

For reviews, see
Giamarchi, *Quantum Physics in One Dimension* (2004)
Cazalilla *et al.*, Rev. Mod. Phys. **83**,1405 (2011)

Tomonaga-Luttinger theory

$$\hat{H}=\frac{\hbar c_s}{2\pi}\int dx\left\{K(\partial_x\hat{\theta})^2+\frac{1}{K}(\partial_x\hat{\phi})^2+V\,n_0\cos\left[\hat{\phi}(x)-\delta x\right]\right\}$$

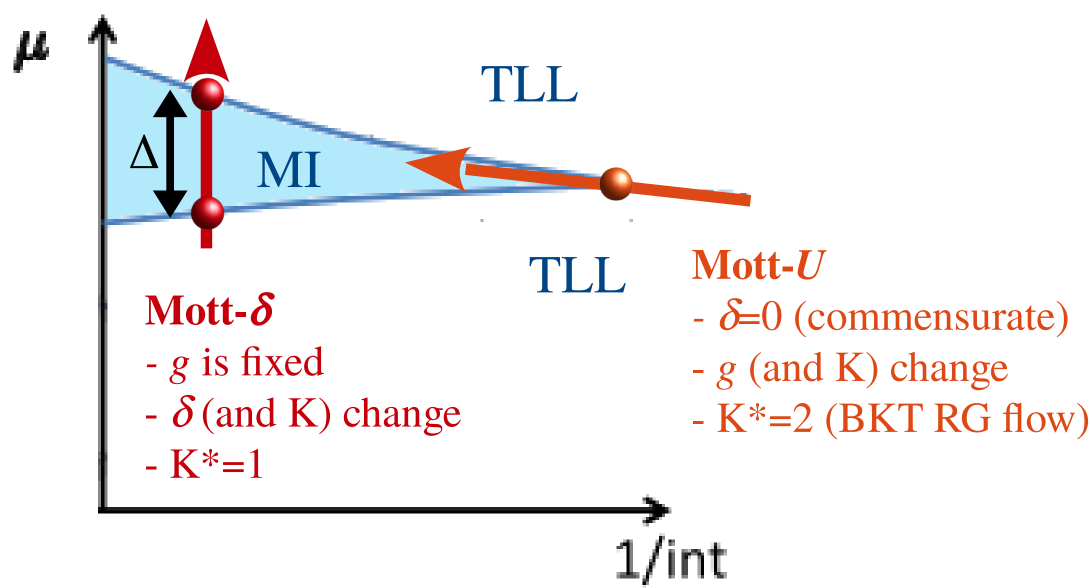
Universal but effective field theory

Unknown parameters (c_s, K, and V)

Renormalization group analysis

Renormalization flow of K, and V

Mott transitions are signaled by the breakdown of TLL theory

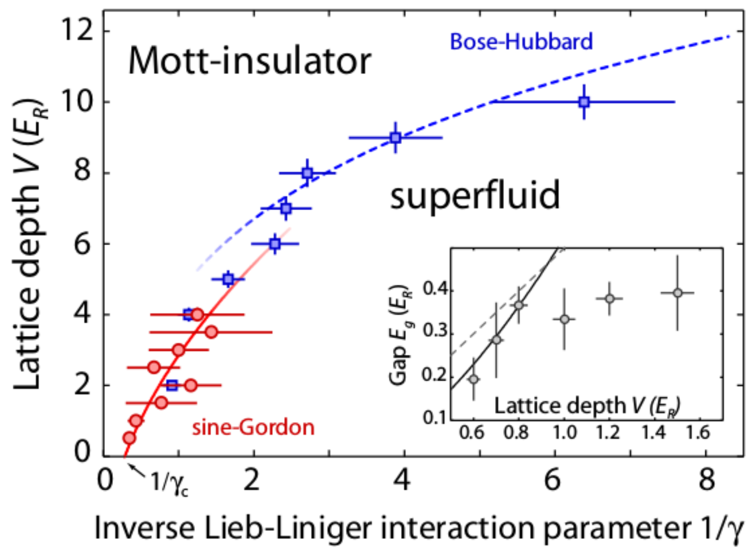


Renormalization Group Analysis within Tomonaga-Luttinger Liquid Theory

Early work

Ultracold atoms in a controlled 1D optical lattice

Observation of the pinning (Mott-U) transition



Haller *et al.*, Nature **466**, 597 (2010)

Open questions

Validity test of the quadratic fluid theory

Mott critical points and quantitative phase diagram?

Mott-δ transition?

Here,

Complementary quantum (UA) and numerical (PIMC) simulations of the full Hamiltonian

$$\hat{H} = \int d\mathbf{r}\; \hat{\Psi}^{\dagger}(\mathbf{r}) \left[\frac{-\hbar^2\nabla^2}{2m} + V(\mathbf{r}) - \mu \right] \hat{\Psi}(\mathbf{r}) + \int d\mathbf{r}d\mathbf{r}'\; \hat{\Psi}^{\dagger}(\mathbf{r})\hat{\Psi}^{\dagger}(\mathbf{r}')U_{\rm int}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}')\hat{\Psi}(\mathbf{r})$$

Continuous space

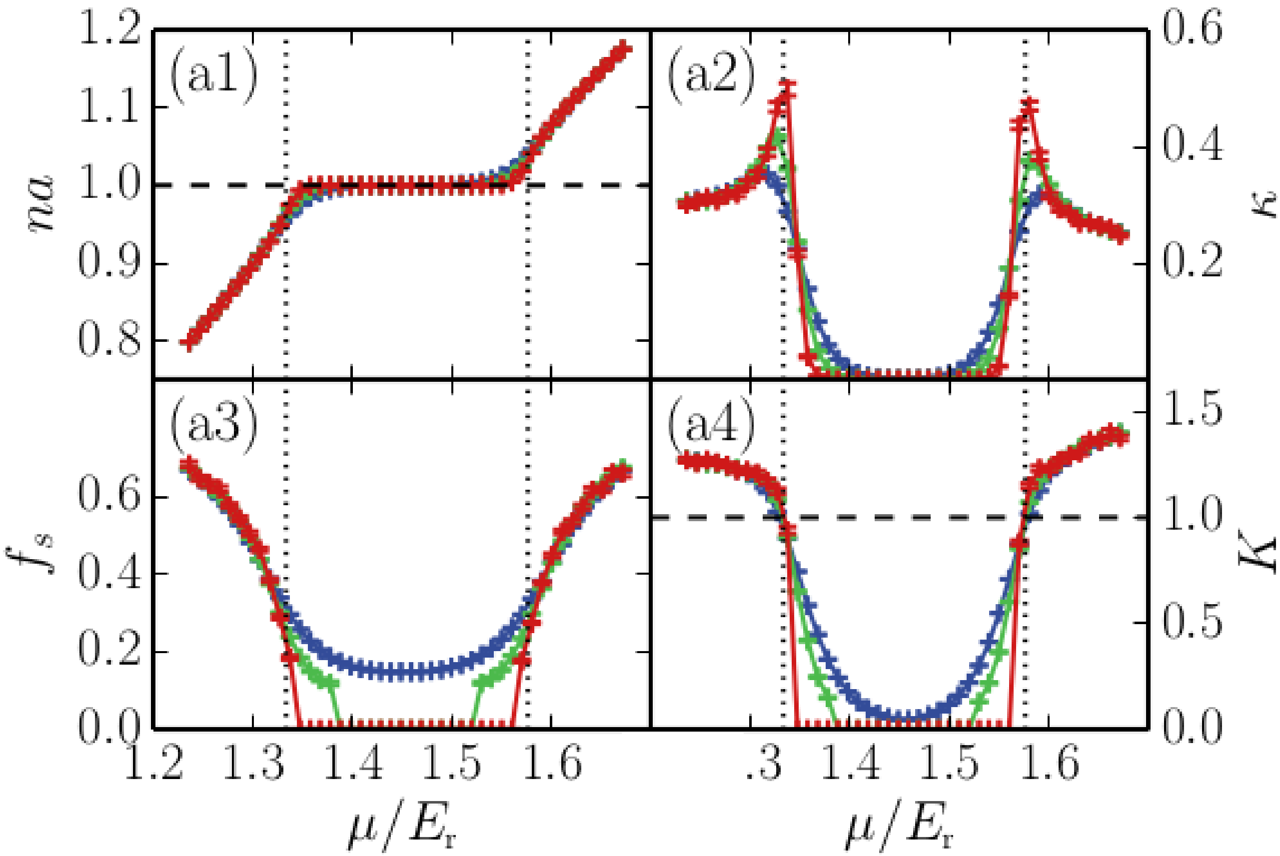
Mott-δ Transition

Quantum path-integral Monte Carlo (PIMC)

density

superfluid fraction

$f_s \equiv \frac{n_s}{n}$



compressibilty

$\kappa \equiv \frac{\partial n}{\partial \mu} = \frac{L \Delta n^2}{k_B T}$

Luttinger parameter

$K = \pi \sqrt{(\hbar^2/m) n_s \kappa}$

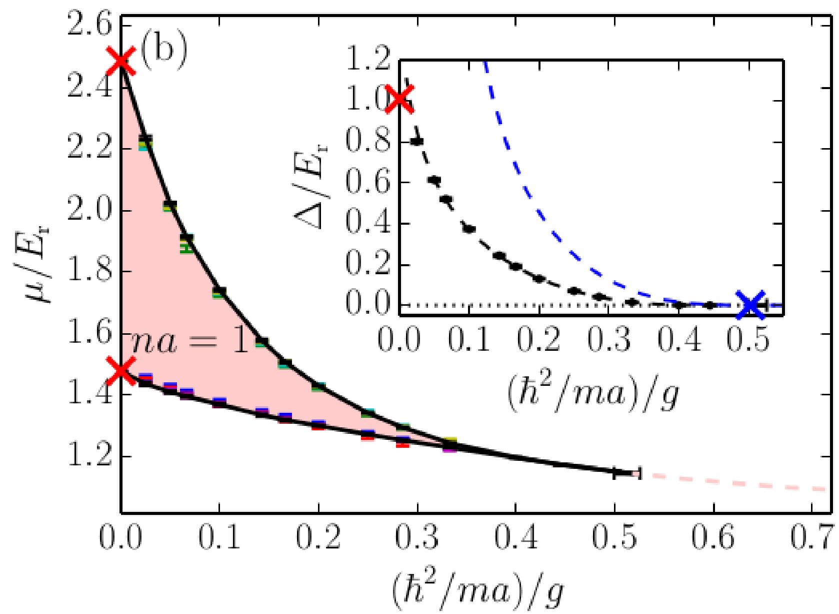
⇨ Provides four independent quantities to find the Mott-δ transition

- (•) Crossing point of the κ curves
- (•) Cusp of the κ curves
- (•) Crossing point K
- (•) K=1

Boéris *et al.*, PRA **93**, 011601(R) (2016)

Quantum Phase Diagram of the Interacting Bose Gas in a Periodic Potential

Quantum path-integral Monte Carlo (PIMC)



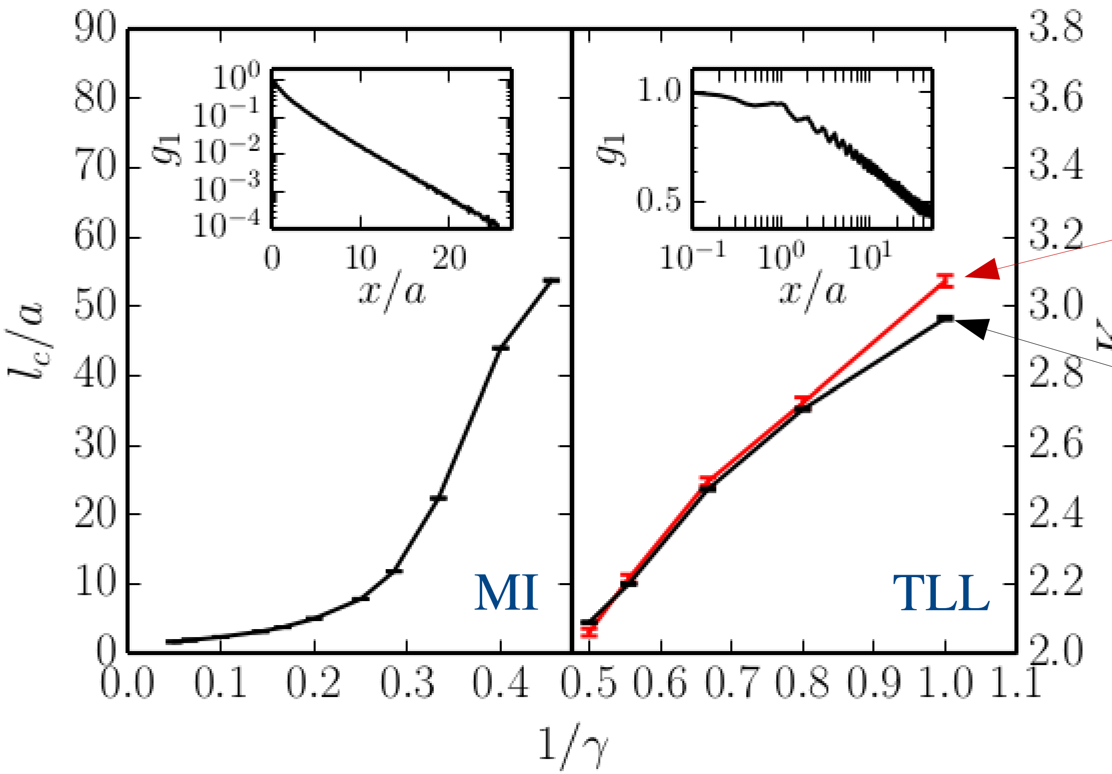
⇨ Universal transition @ $K^*=1$
⇨ Quantitatve phase diagram
⇨ Exponential shape of gap up to $g \sim 100\,\hbar^2/ma$

Boéris *et al.*, PRA **93**, 011601(R) (2016)

Mott-*U* Transition

Quantum path-integral Monte Carlo (PIMC)

$$g_1(x) = \langle \hat{\Psi}^\dagger(x) \hat{\Psi}(0) \rangle$$



$K = \pi\sqrt{(\hbar^2/m) n_s \kappa}$

$g_1(x) \sim 1/x^{1/2K}$

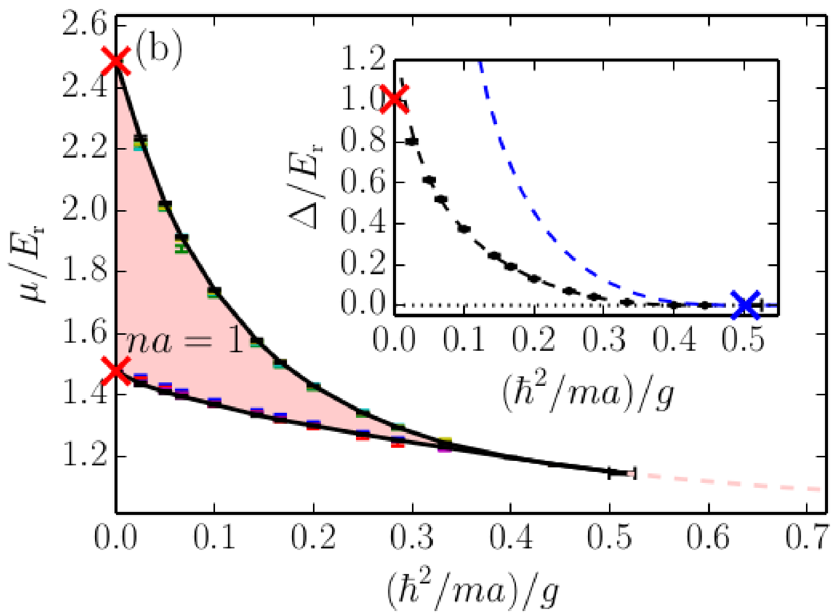
$K \simeq 2$

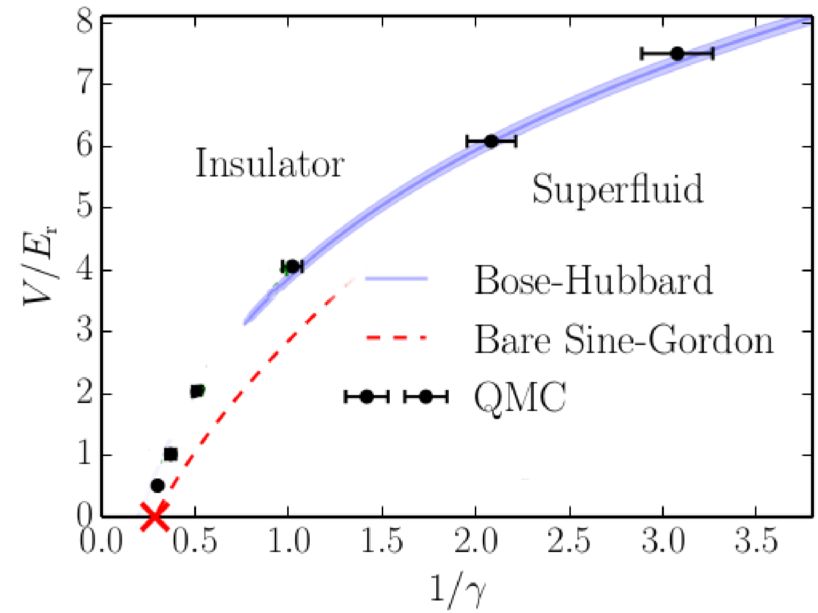
$g_c(V)$

Boéris *et al.*, PRA **93**, 011601(R) (2016)

Quantum Phase Diagram of the Interacting Bose Gas in a Periodic Potential

Quantum path-integral Monte Carlo (PIMC)





⇨ Universal transition @ $K^*=1$
⇨ Quantitatve phase diagram
⇨ Exponential shape of gap up to $g \sim 100\,\hbar^2/ma$

⇨ BKT transition @ $K^*=2$
⇨ Quantitatve phase diagram
⇨ Significant deviation from unrenormalized field theory

Boéris *et al.*, PRA **93**, 011601(R) (2016)
Astrakharchik *et al.*, PRA **93**, 012605(R) (2016)

Mott-*U* Transition : Experiments

Experimental setup

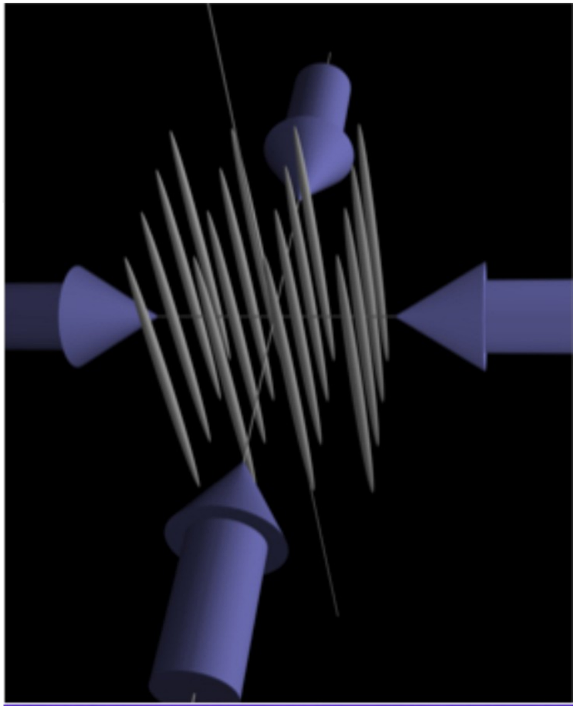
BEC in a 3D harmonic trap with ~35000 ^{39}K atoms

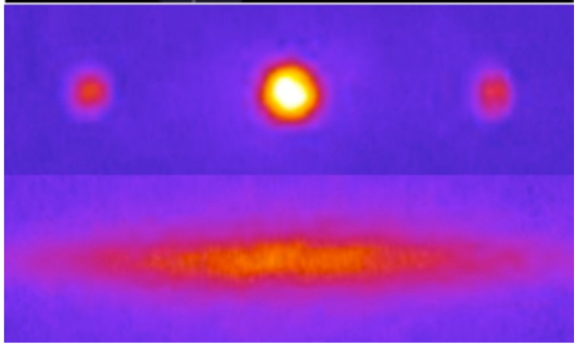
Adiabatic spitting into ~1000 1D tubes (strong 2D optical lattice) ; ~35 atoms per tube

Magnetic levitation

Adiabatic raise of a weak 1D optical lattice along the tubes

Tunable interactions (broad Fano-Feshbach resonance) :
$0.07 \leqslant \gamma \leqslant 7.4$





Boéris *et al.*, PRA **93**, 011601(R) (2016)

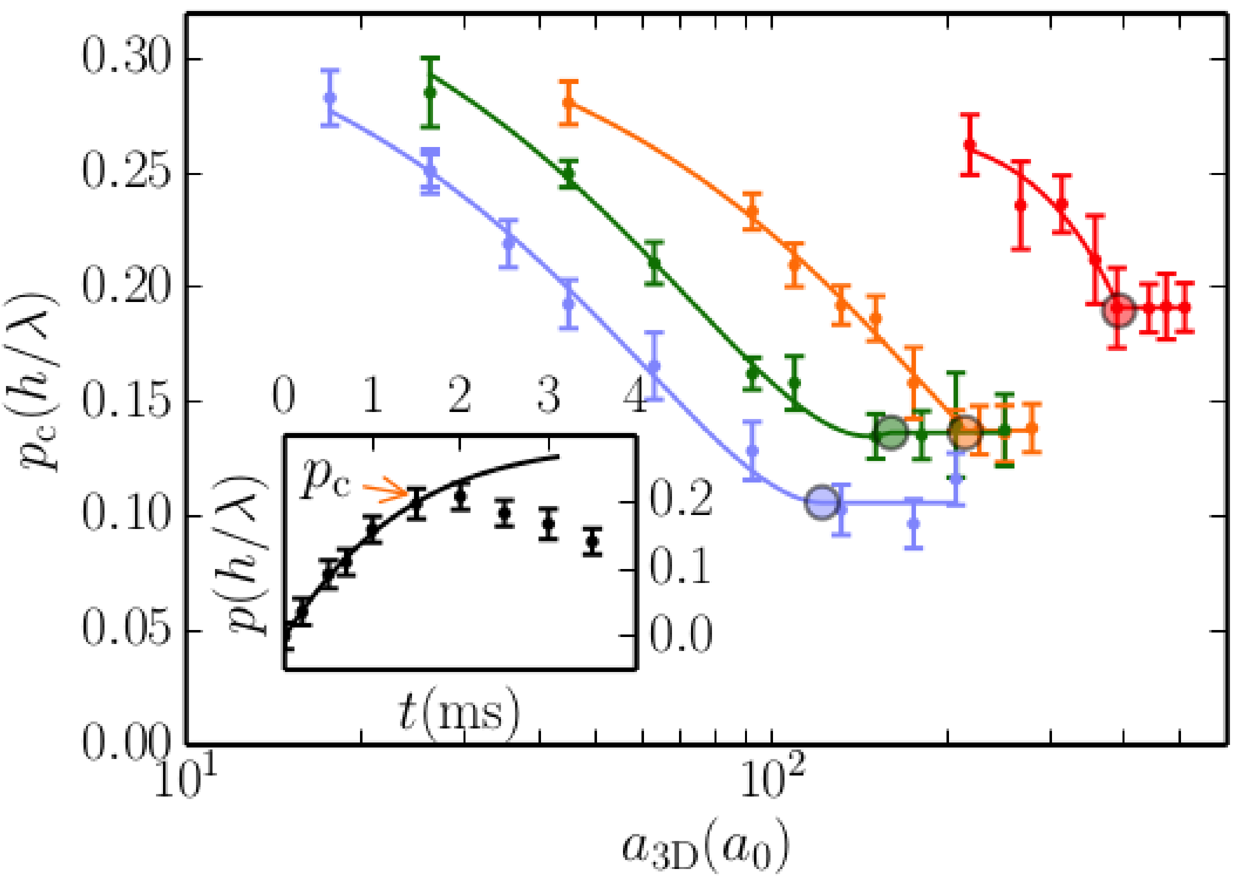
Mott-U Transition : Experiments

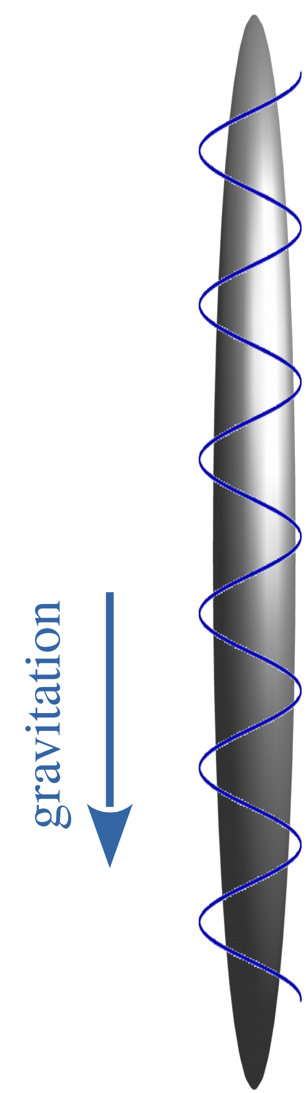
Transport measurement

Switch off levitation

Acceleration of the atoms for a time t

Measurement of momentum p



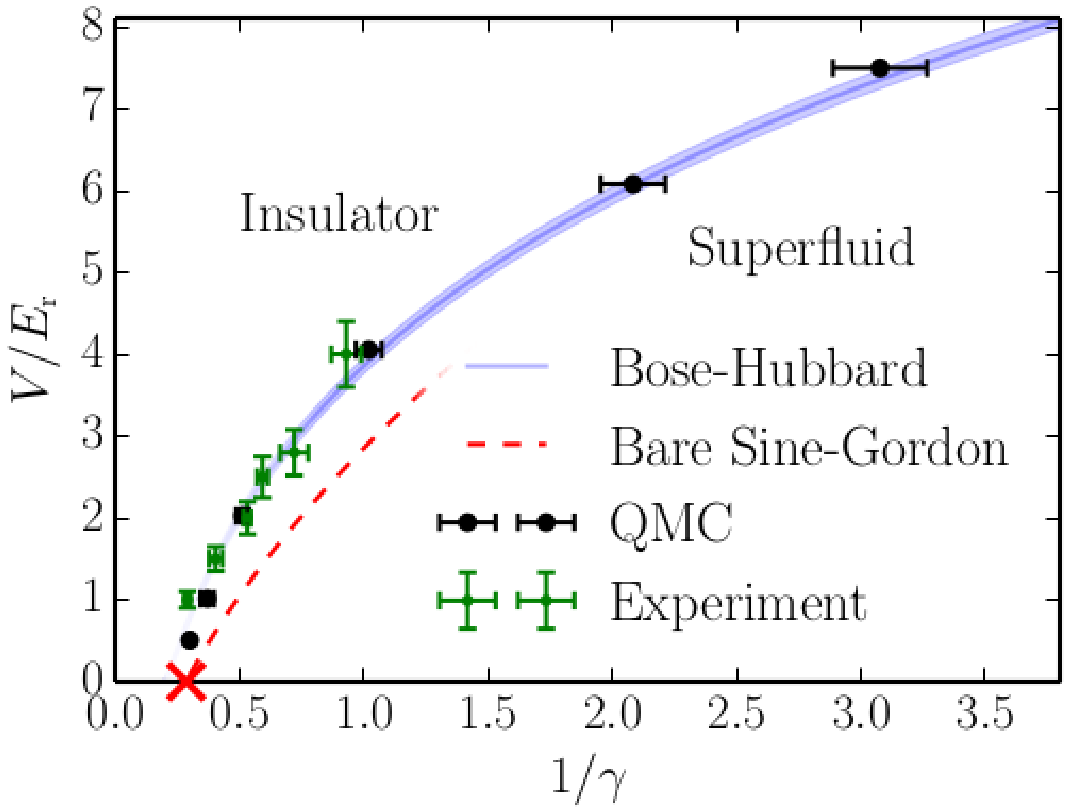


Boéris *et al.*, PRA **93**, 011601(R) (2016)

Mott-U Transition : Experiments

Boéris *et al.*, PRA **93**, 011601(R) (2016)

Transport measurement



⇨ Accurate determination of critical parameters
⇨ Excellent agreement with theory
⇨ Validation of a quantum simulator

Boéris *et al.*, PRA **93**, 011601(R) (2016)

Perspectives

Higher dimensions

Are there critical values of the interaction and/or periodic potentials in 2D and 3D?

Numerically much harder ….

Disordered systems and Bose-glass transitions

Disorder or quasi-disorder can be controlled

Bose-glass transition expected but never observed

Critical behavior, in particular for low interactions

Bose-glass versus Mott transition in bichromatic lattices

Long-range interactions

Mott transition with fractional filling

Droplet phase beyond modified mean field theory

Interplay of (quasi-)disorder and long-range interactions (Bose-glass physics, MBL, ...)