

# Quantum multimode resources based on optical frequency combs and implementation of quantum complex networks

Valentina Parigi

Nicolas Treps, Claude Fabre

Multimode quantum optics group

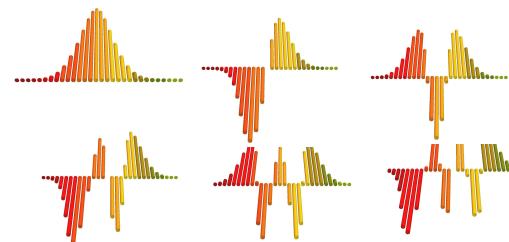


20 October 2017



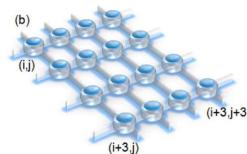
## Multimode quantum optics

multipartite entanglement

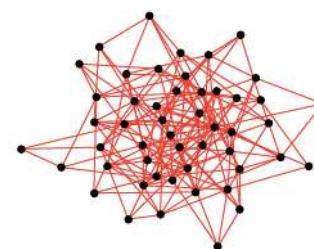


## Study of quantum network

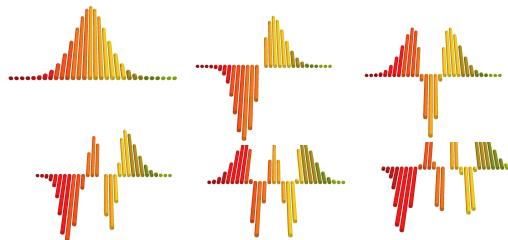
cluster states



quantum complex networks



## Multimode quantum optics multipartite entanglement

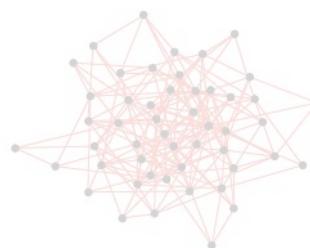


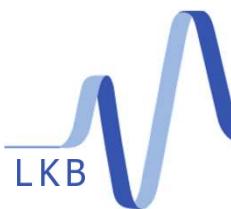
## Study of quantum network

cluster states



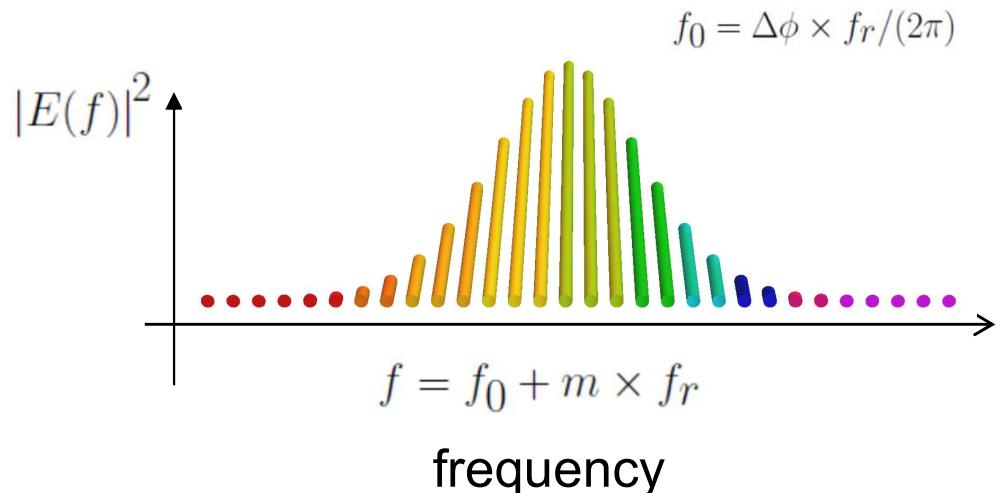
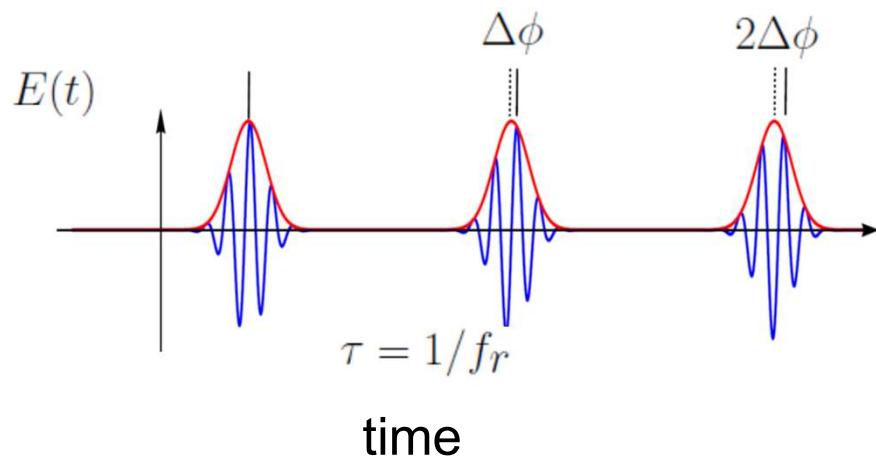
quantum complex networks



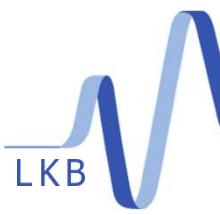


# Multimode quantum optics

Optical frequency comb + quantum optics

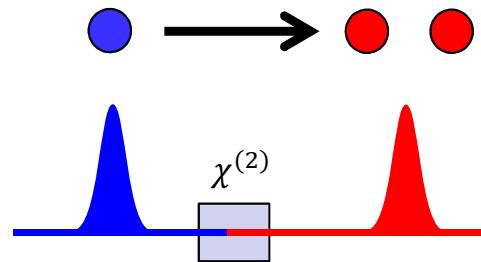
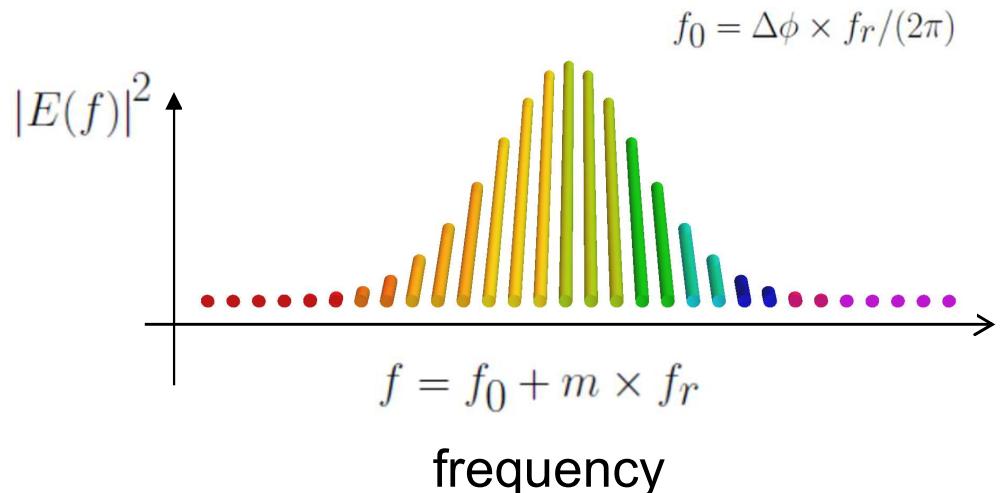
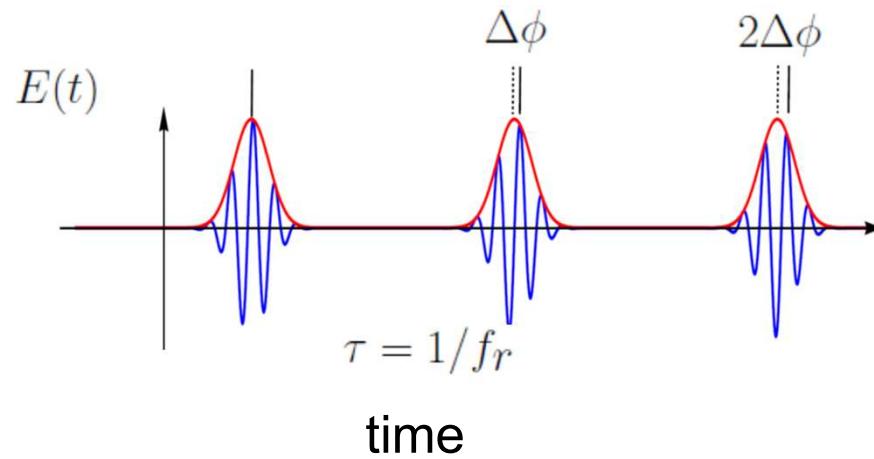


Femtosecond mode-locked laser  $\rightarrow$  Huge number of modes ( $\sim 10^6$  frequencies)



# Multimode quantum optics

Optical frequency comb + **quantum optics**



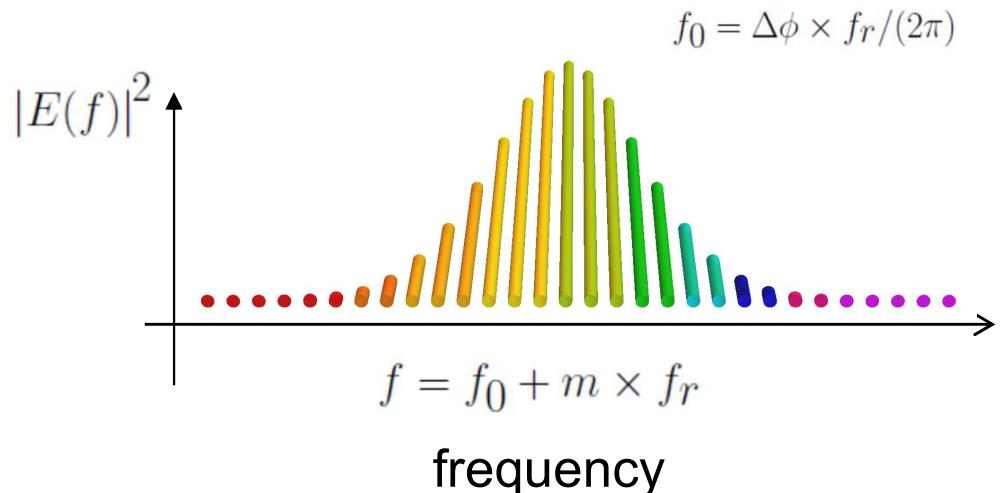
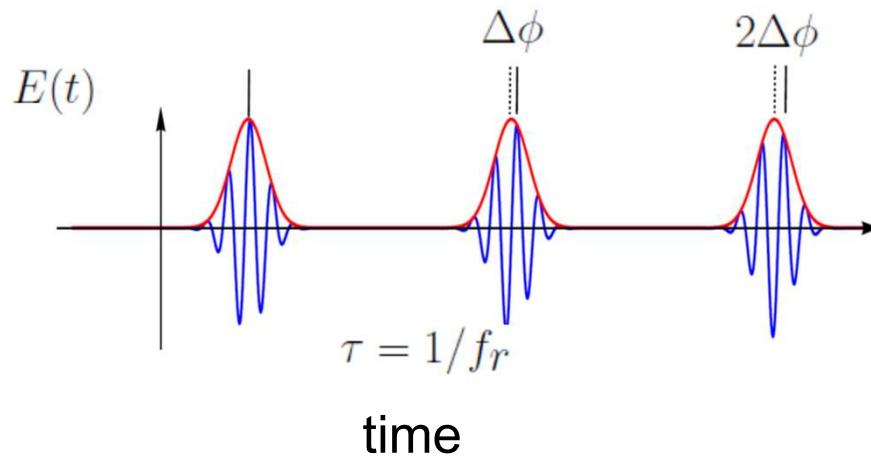
$$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$$

Parametric process

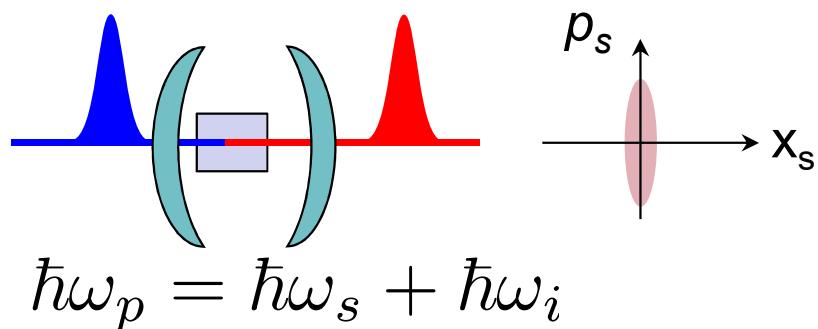


# Multimode quantum optics

Optical frequency comb + **quantum optics**



$$\omega_s = \omega_i$$



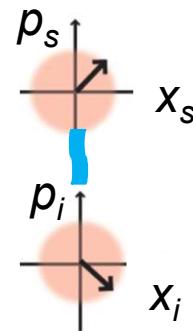
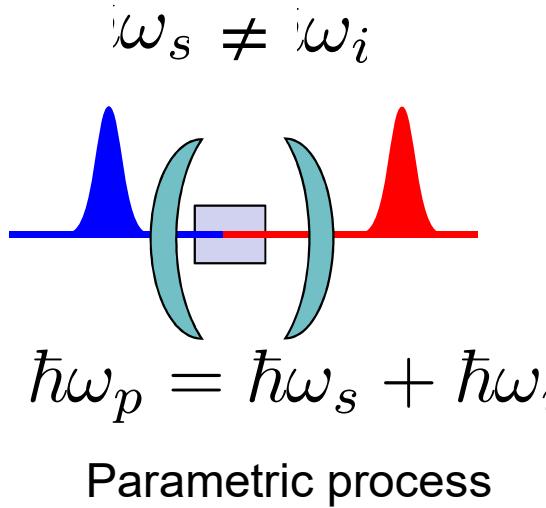
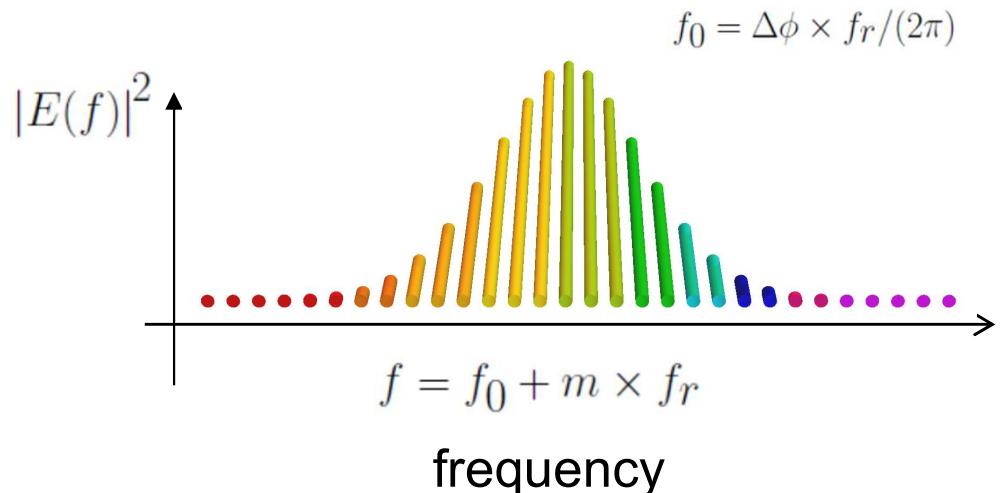
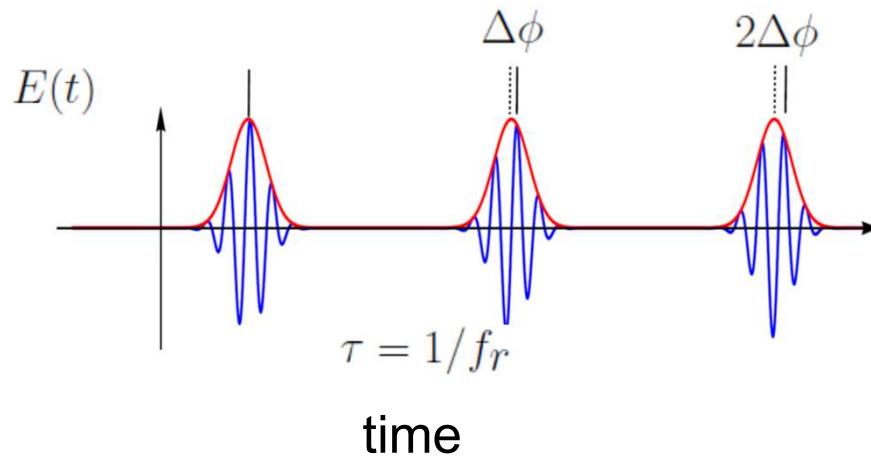
Parametric process

squeezed states  
Variance of  $p <$  shot noise



# Multimode quantum optics

Optical frequency comb + **quantum optics**



Quadrature entangled states

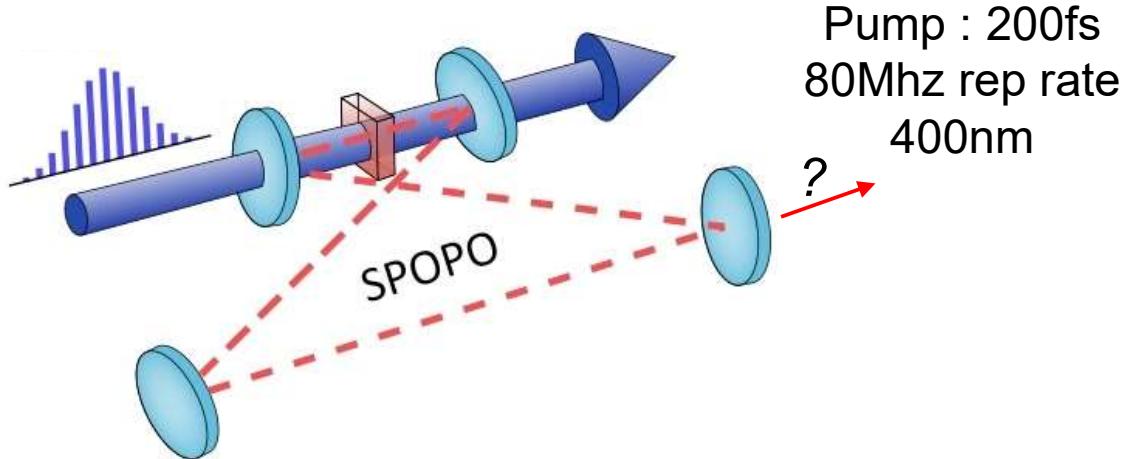
$x_s$  et  $x_i$  correlated  
 $p_s$  et  $p_i$  anti-correlated

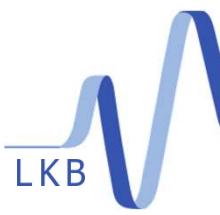
Parametric process



## Optical frequency comb + quantum optics

### Synchronously pumped OPO

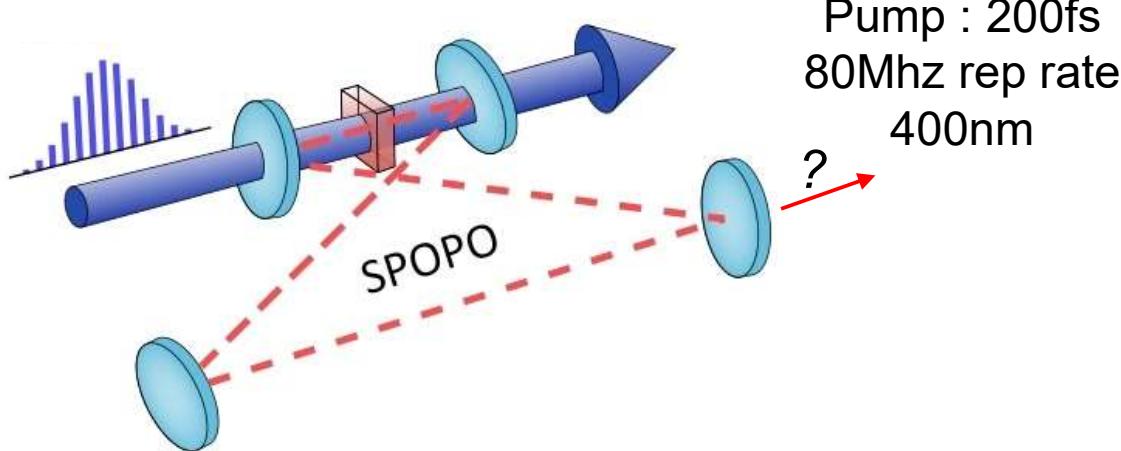




# Multimode quantum optics

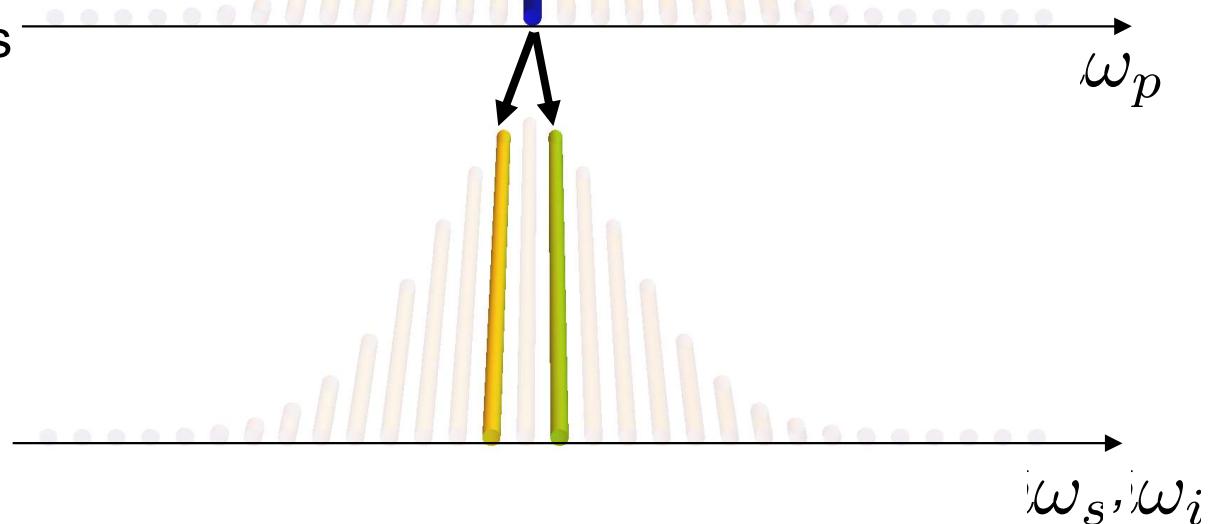
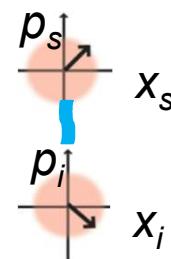
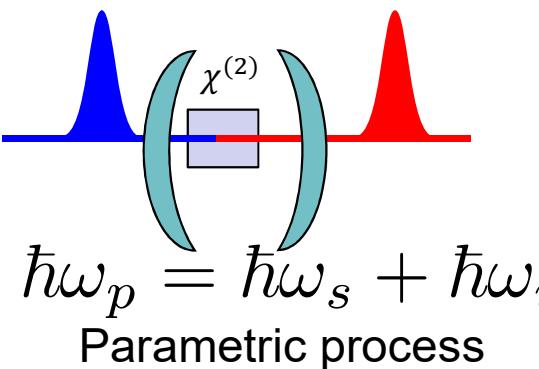
## Optical frequency comb + quantum optics

### Synchronously pumped OPO



$$H = i\hbar \sum_{m,n} L_{-m,n} \hat{a}_{-m}^\dagger \hat{a}_n^\dagger$$

### Symmetric Frequency Correlations (quadratures entanglement)

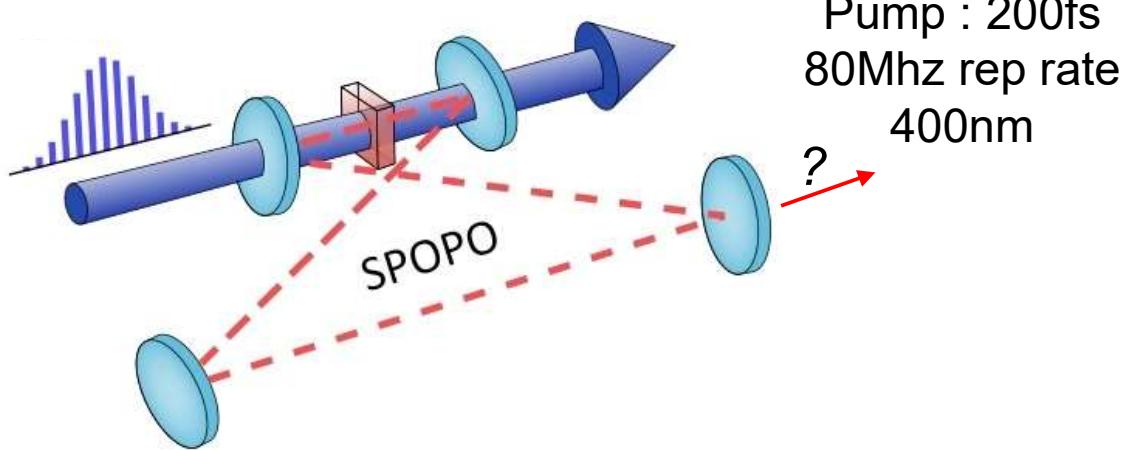




# Multimode quantum optics

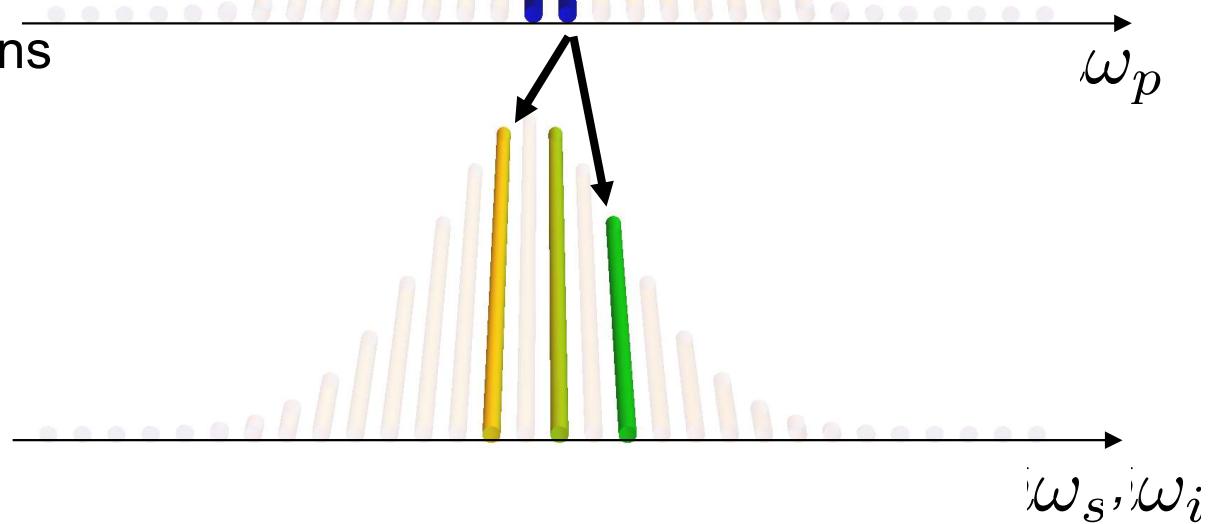
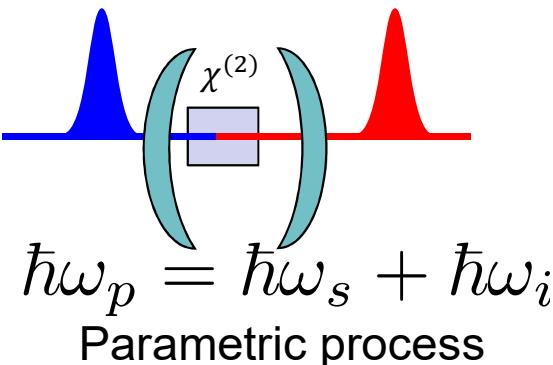
## Optical frequency comb + quantum optics

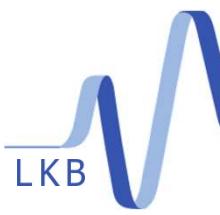
### Synchronously pumped OPO



$$H = i\hbar \sum_{m,n} L_{-m,n} \hat{a}_{-m}^\dagger \hat{a}_n^\dagger$$

### Asymmetric Frequency Correlations (quadratures entanglement)

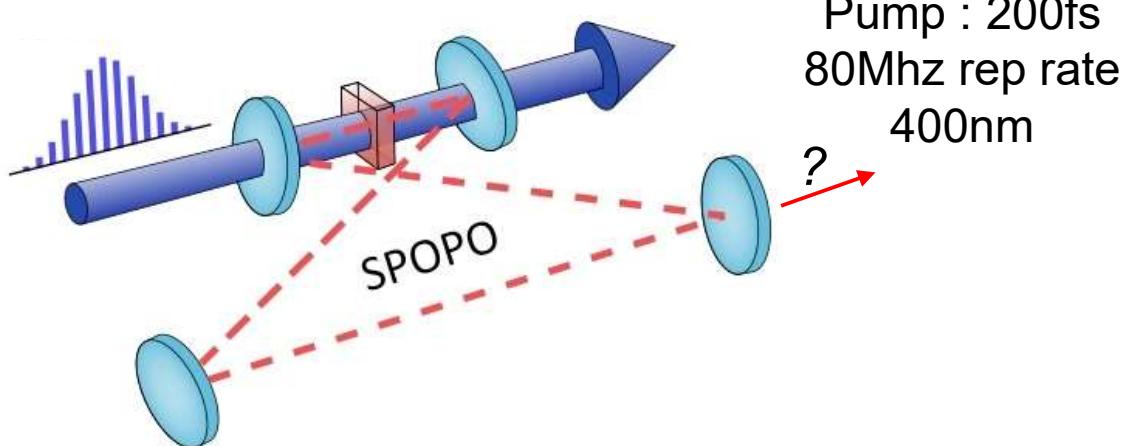




# Multimode quantum optics

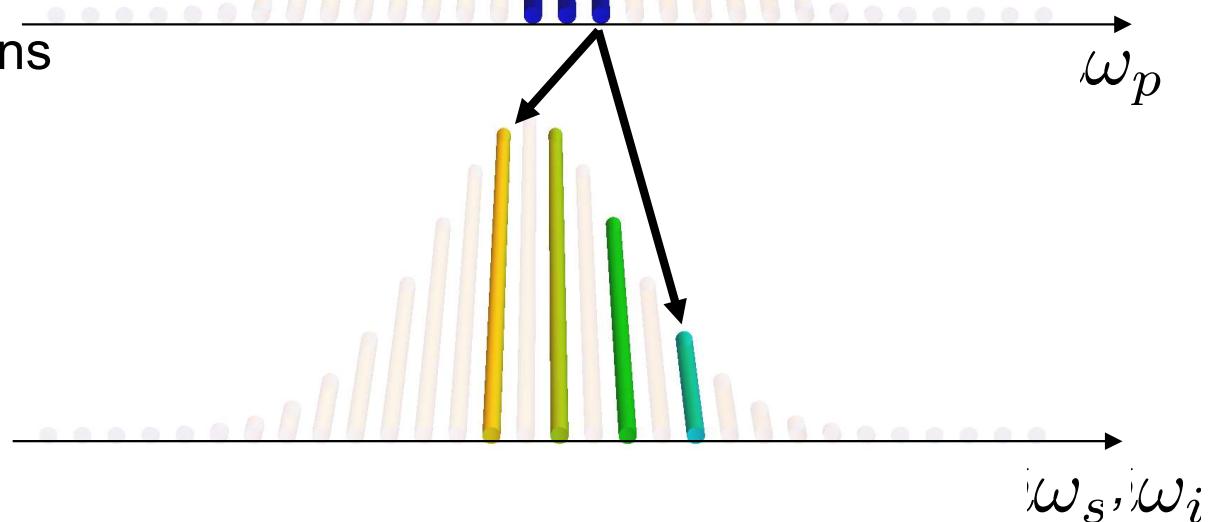
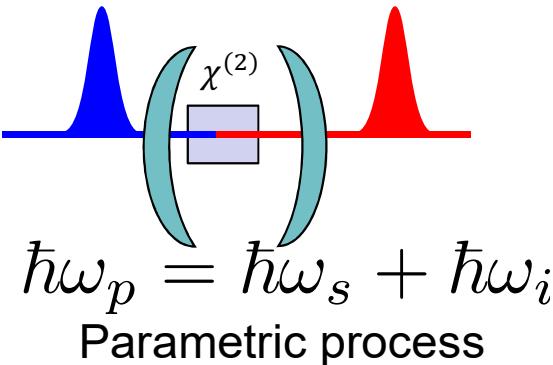
## Optical frequency comb + quantum optics

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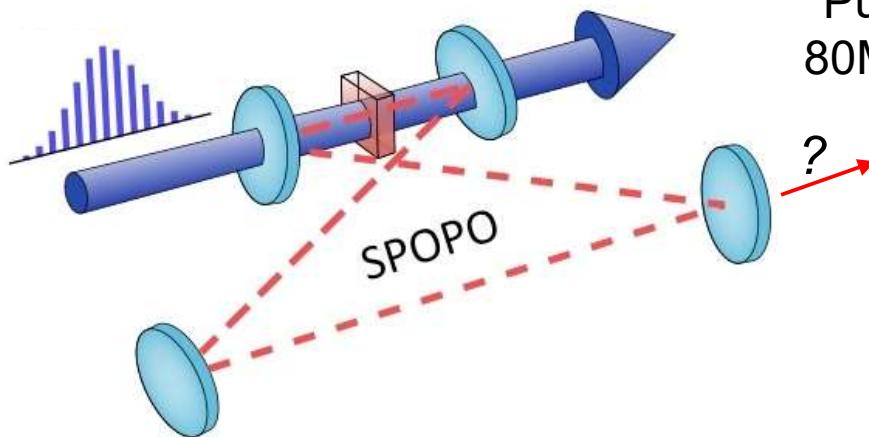




# Multimode quantum optics

## Optical frequency comb + quantum optics

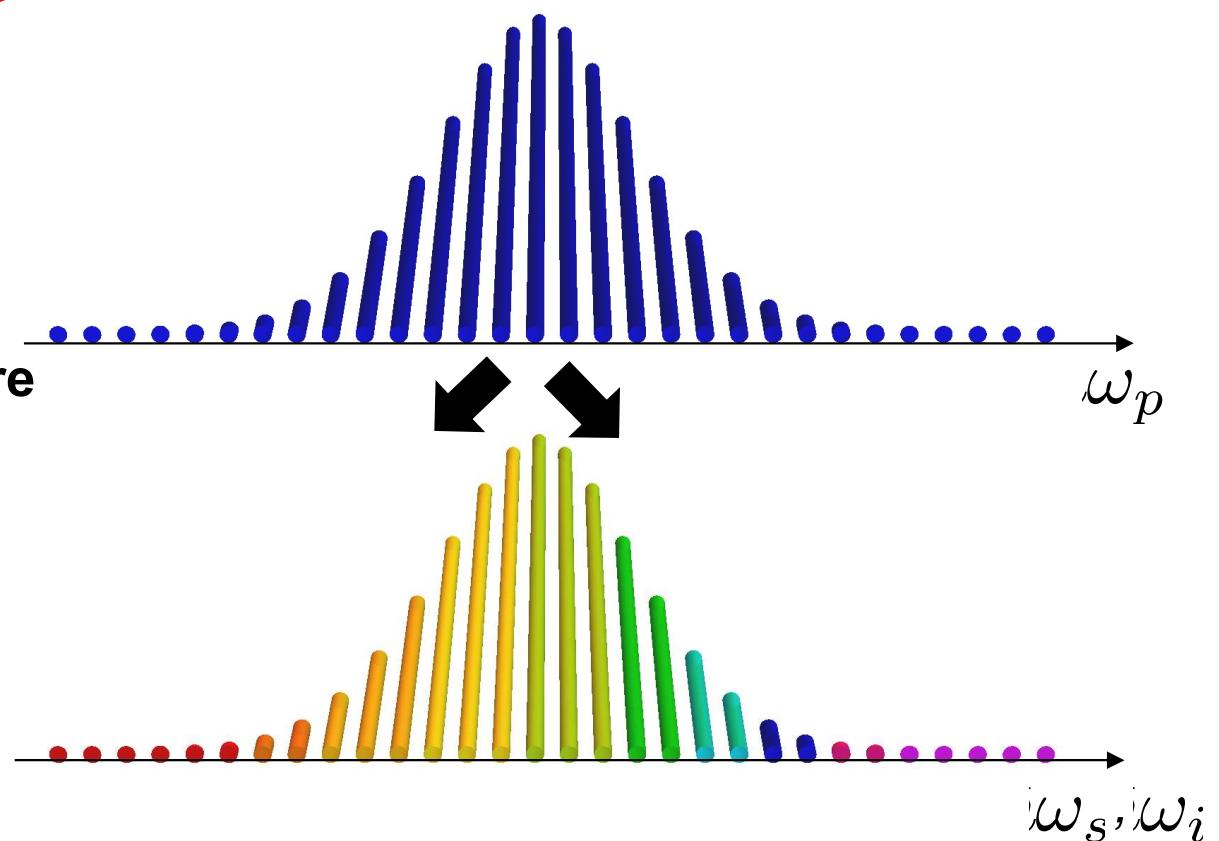
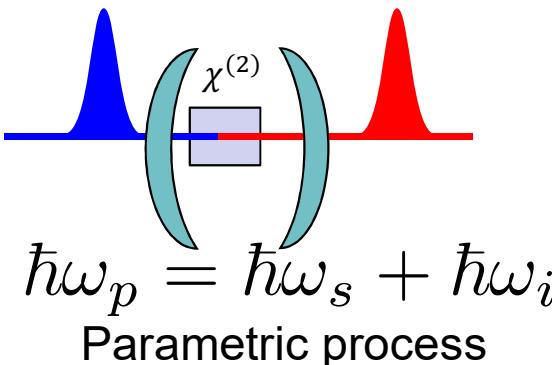
### Synchronously pumped OPO



Pump : 200fs  
80Mhz rep rate  
400nm

$$H = i\hbar \sum_{m,n} L_{-m,n} \hat{a}_{-m}^\dagger \hat{a}_n^\dagger$$

### Non-trivial entanglement structure (quadratures entanglement)

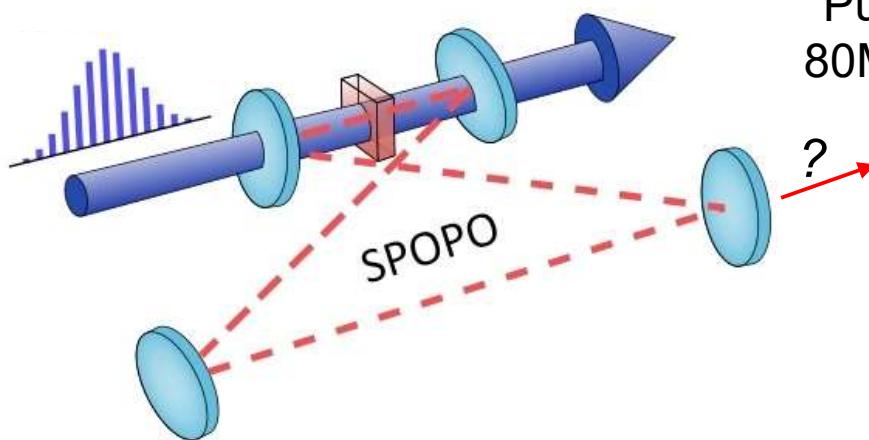




# Multimode quantum optics

## Optical frequency comb + quantum optics

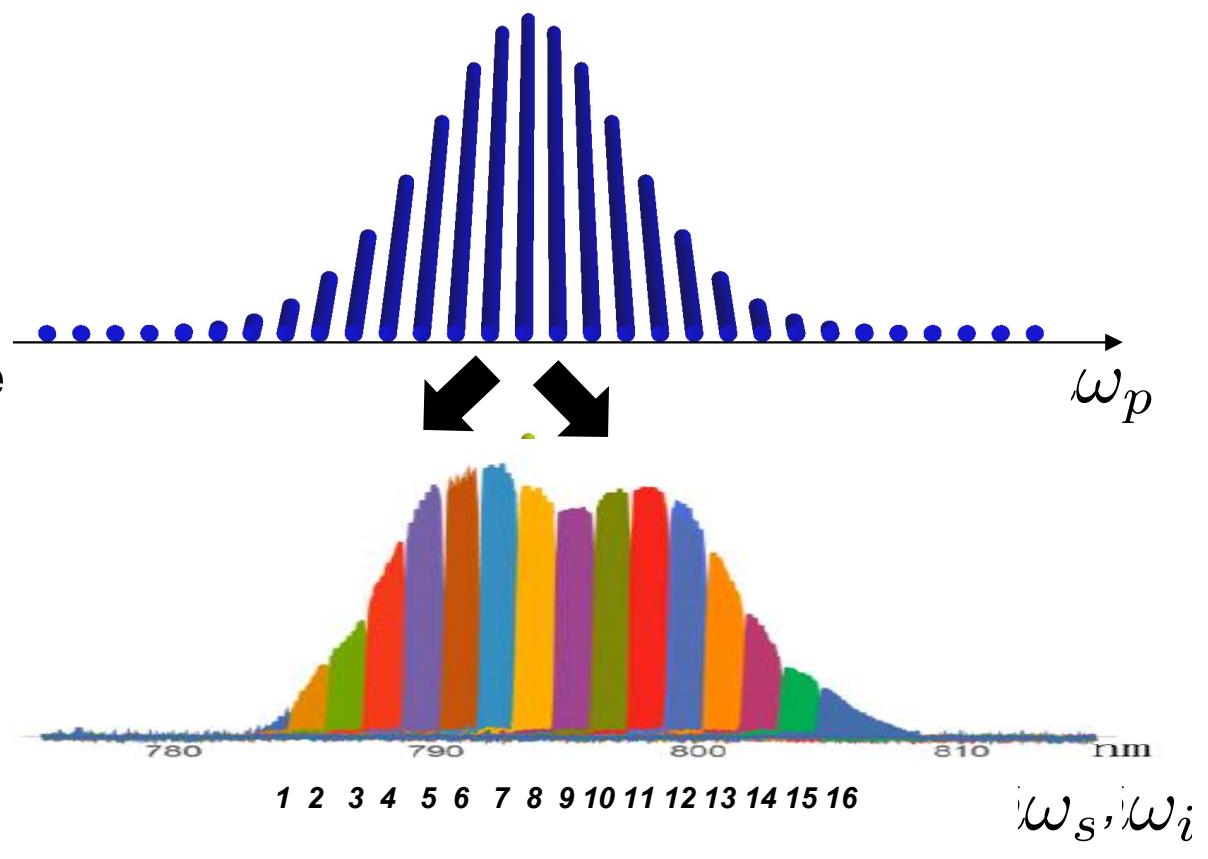
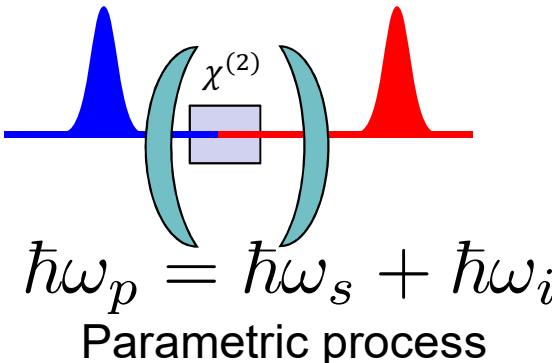
### Synchronously pumped OPO



Pump : 200fs  
80Mhz rep rate  
400nm

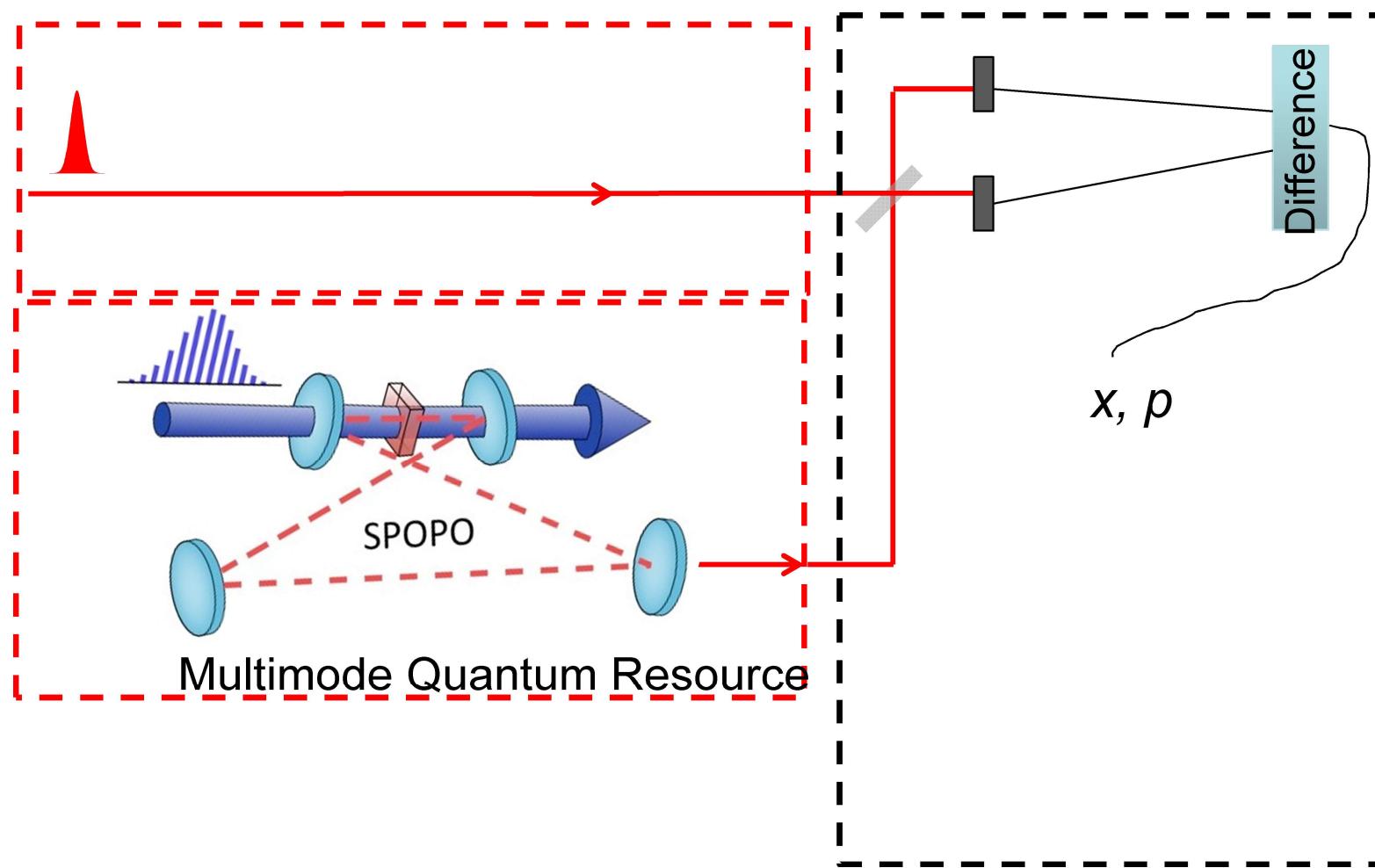
$$H = i\hbar \sum_{m,n} L_{-m,n} \hat{a}_{-m}^\dagger \hat{a}_n^\dagger$$

### Complex entanglement structure (quadratures entanglement)

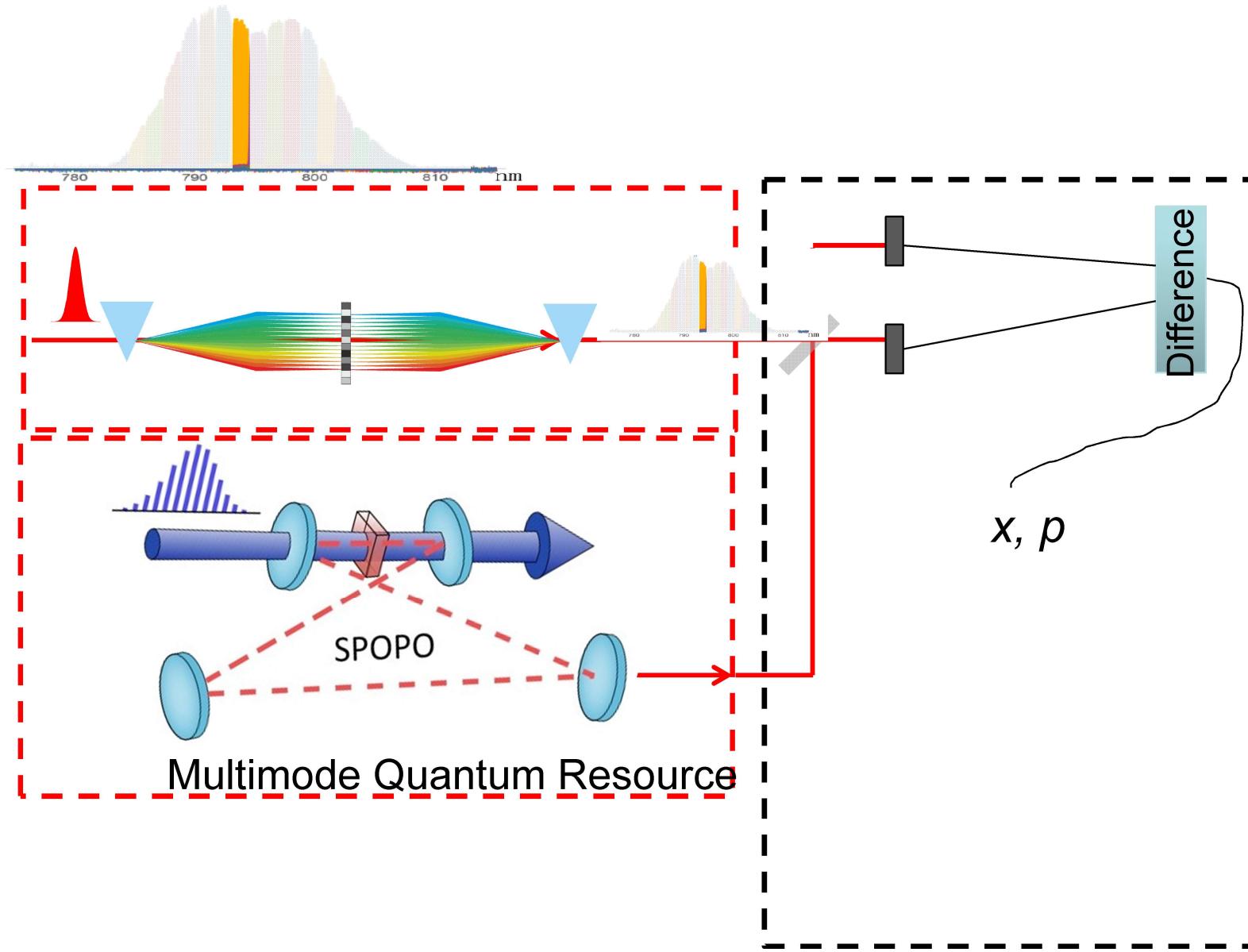


Characterization :

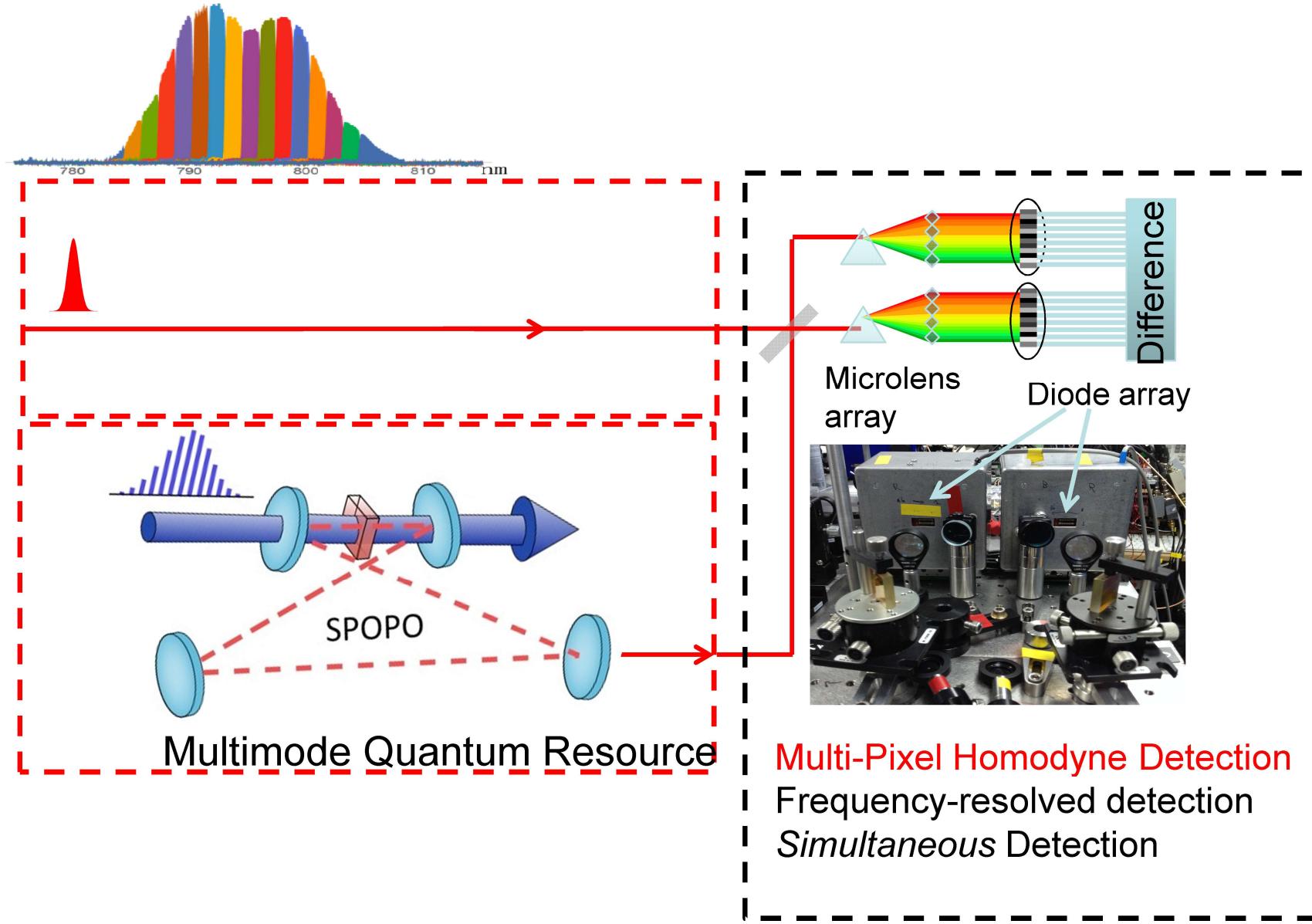
homodyne detection

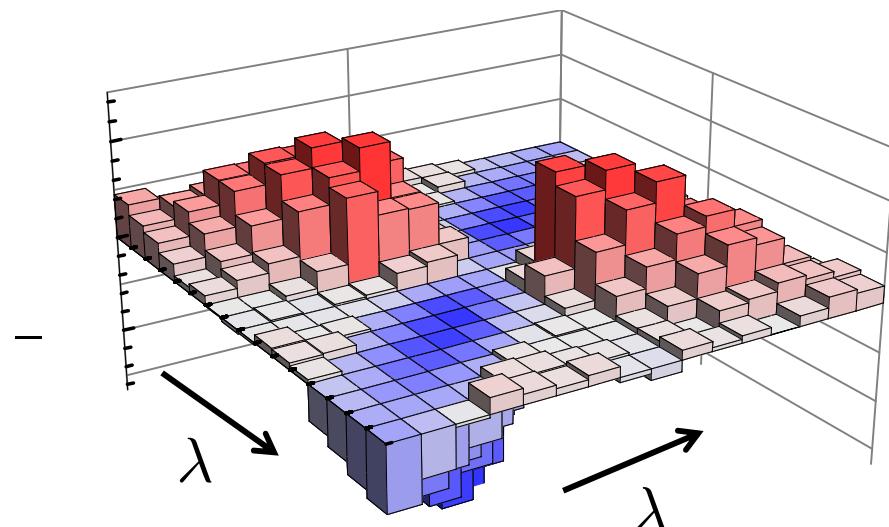
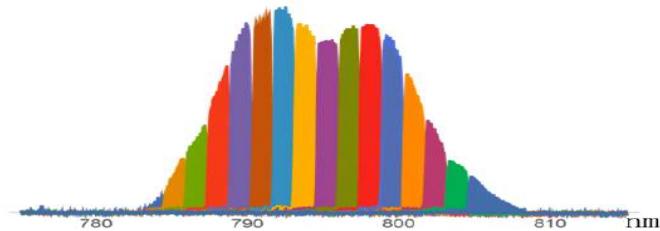


## Characterization : mode-selective homodyne detection

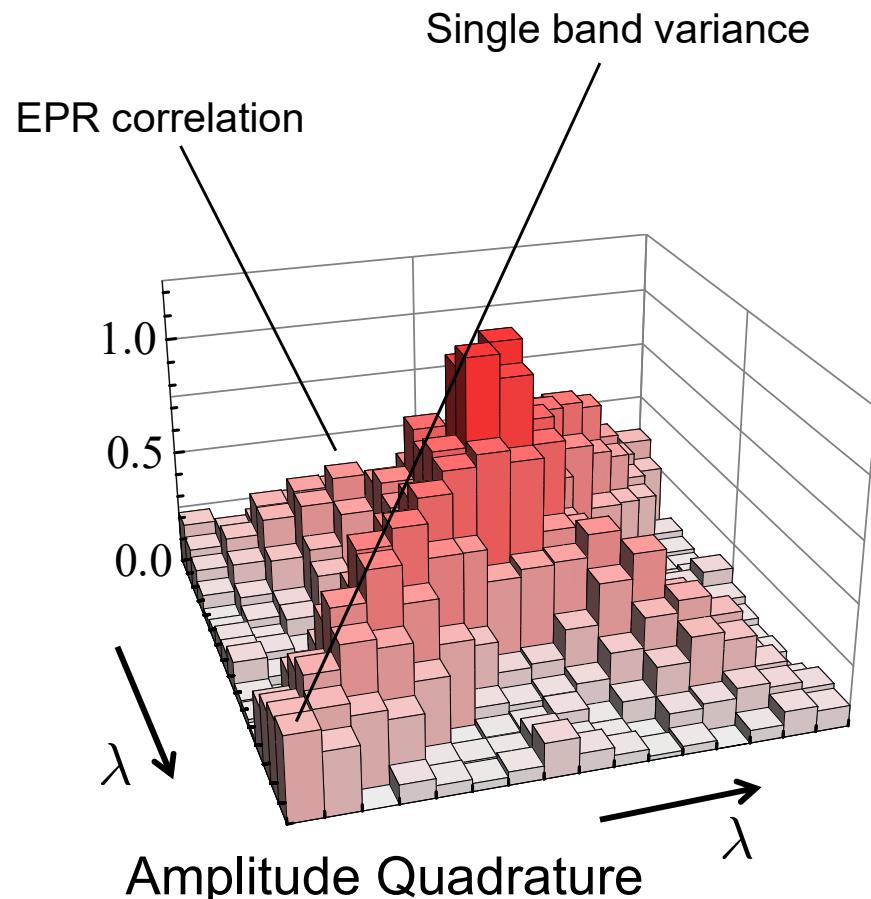


## Characterization : multi-mode homodyne detection





16-mode Covariance matrix  
of Phase Quadrature

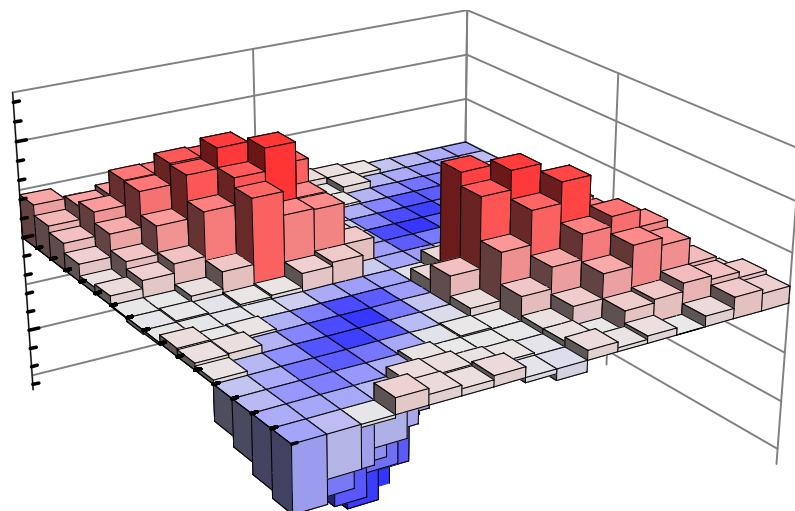


$$\frac{\langle \delta\hat{q}_i\delta\hat{q}_j \rangle + \langle \delta\hat{q}_j\delta\hat{q}_i \rangle}{2}$$

## Diagonalization



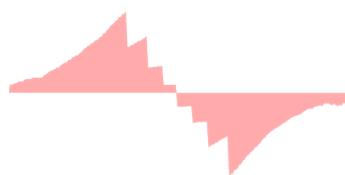
- Independent Squeezers
- Unique temporal / spectral pulse shapes



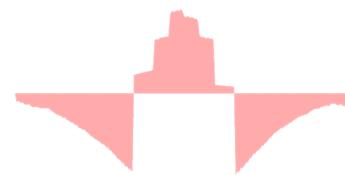
J. Roslund, R. Medeiros, S. Jiang,  
 C. Fabre and N. Treps,  
 Nature Photonics 8, 109–112 (2014)



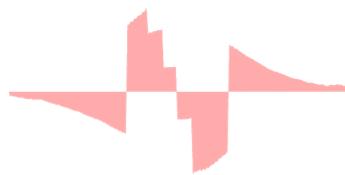
**-4.6dB**



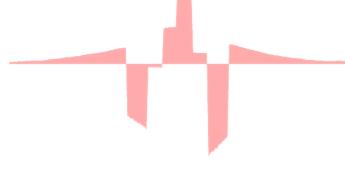
**-3.5dB**



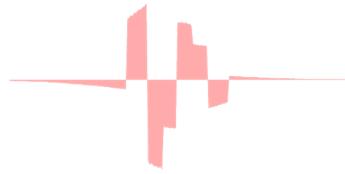
**-1.7dB**



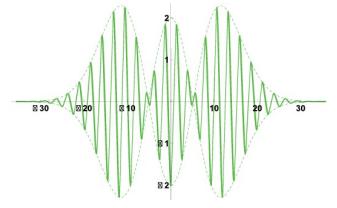
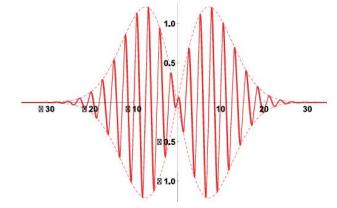
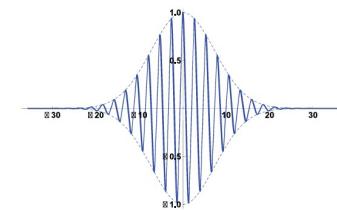
**-1.5dB**



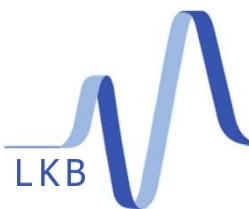
**-0.8dB**



**-0.5dB**



■ ■ ■



# Eigenmodes

Spectrum · Noise level · Pulse shape

## Diagonalization



- Independent Squeezers
- Unique temporal / spectral pulse shapes = «supermodes»

It corresponds to

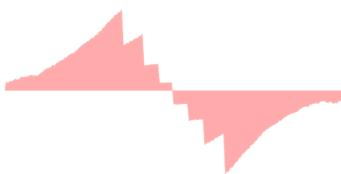
$$H = i\hbar \sum_{m,n} L_{-m,n} \hat{a}_{-m}^\dagger \hat{a}_n^\dagger$$



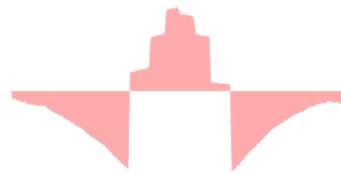
$$H = i\hbar \sum_k \Lambda_k \hat{S}_k^{\dagger 2}$$



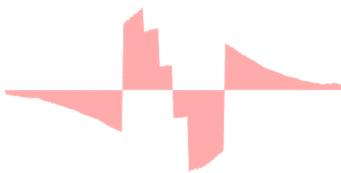
-4.6dB



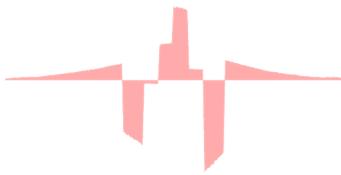
-3.5dB



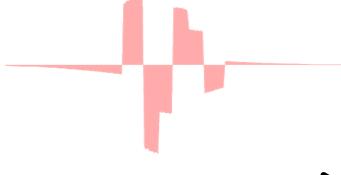
-1.7dB



-1.5dB

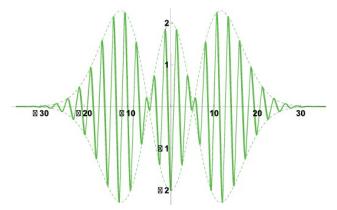
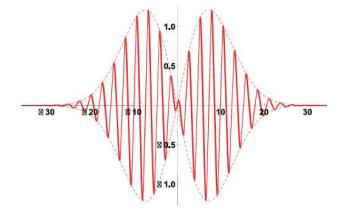
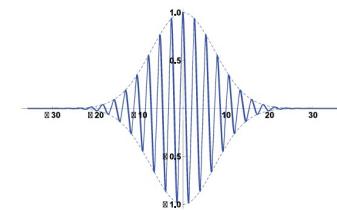


-0.8dB



-0.5dB

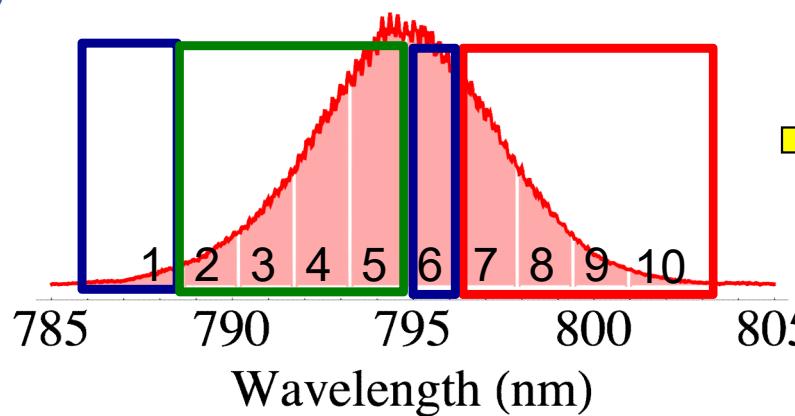
Wavelength



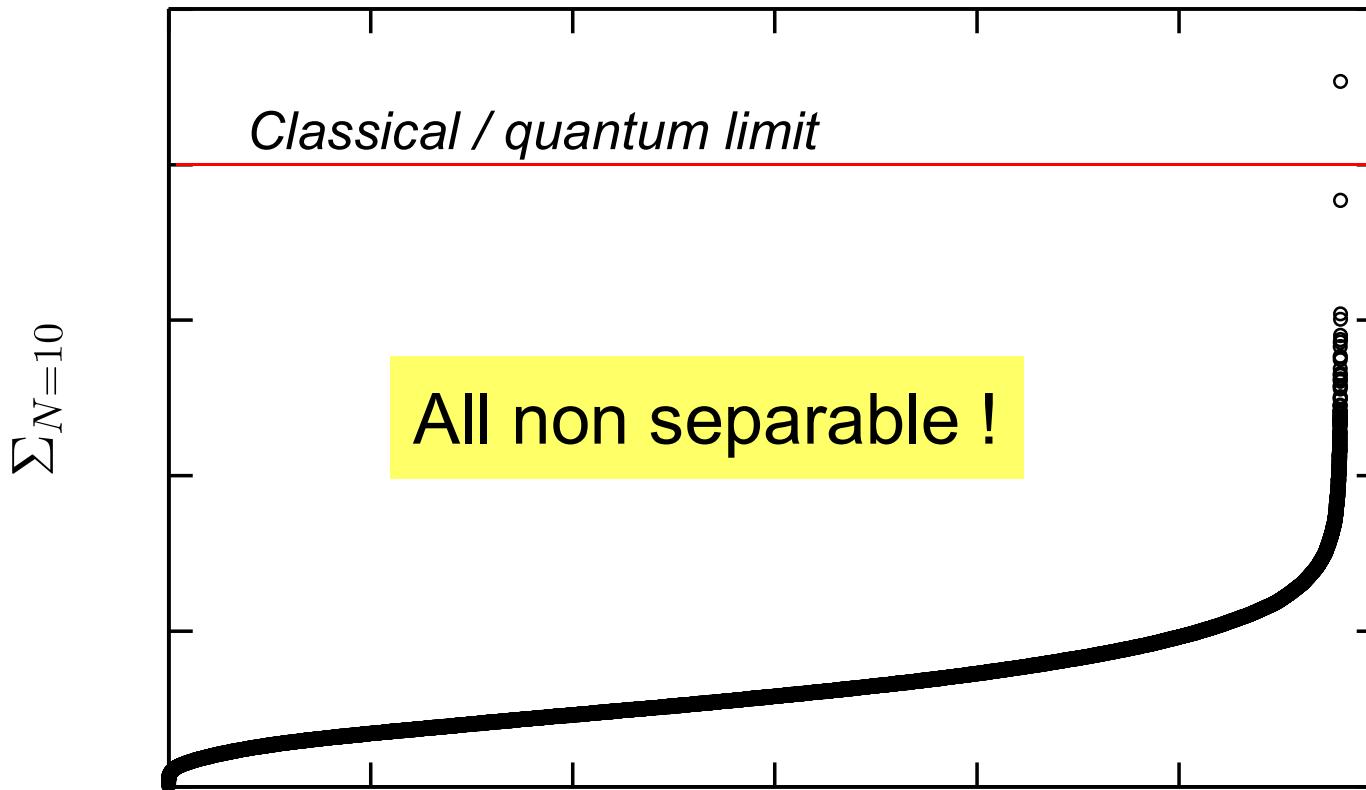
■  
■  
■



# Multimode entangled state



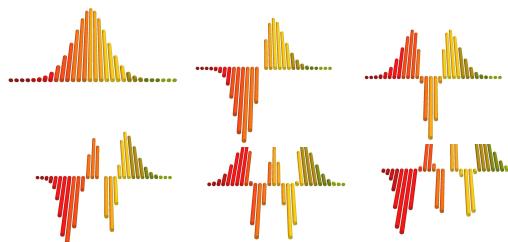
10 frequency bands  
115 974 possible partitions



S. Gerke, J. Sperling, W. Vogel, Y. Cai, J. Roslund, N. Treps, and C. Fabre, Phys Rev Lett **114**, 050501 (2015).

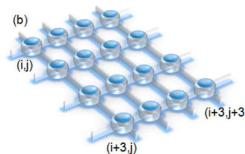
## Multimode quantum optics

multipartite entanglement

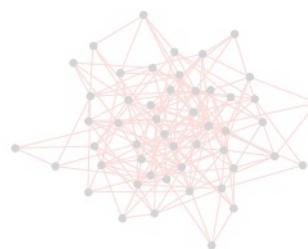


## Study of quantum network

cluster states



quantum complex networks





# Bloch-Messiah decomposition

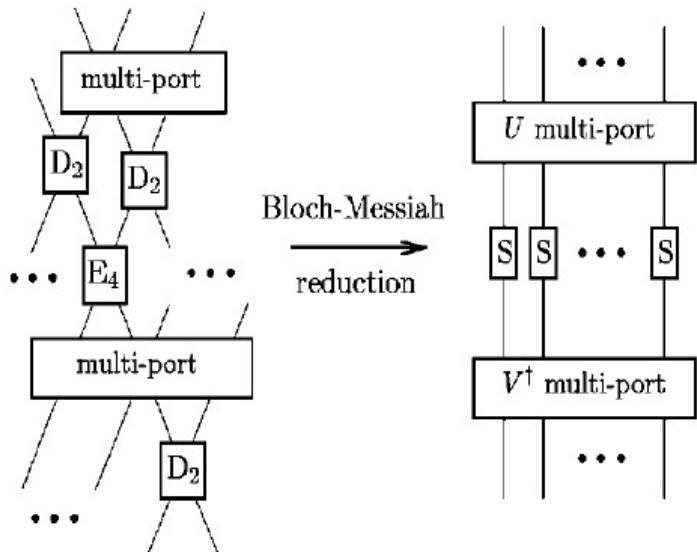
PHYSICAL REVIEW A 71, 055801 (2005)

## Squeezing as an irreducible resource

Samuel L. Braunstein

Computer Science, University of York, York YO10 5DD, United Kingdom

(Received 6 March 2005; published 31 May 2005)



$$\begin{pmatrix} \mathbf{q}_{out} \\ \mathbf{p}_{out} \end{pmatrix} = \mathbf{S} \begin{pmatrix} \mathbf{q}_{in} \\ \mathbf{p}_{in} \end{pmatrix}$$

multimode quantum Gaussian ressource = Squeezed modes + basis change

$$\vec{a} = (a_1, a_2, \dots, a_N).$$

*collection of input modes*

$$\{\mathbf{q}, \mathbf{p}\} = (q_1, q_2, \dots, q_N, p_1, p_2, \dots, p_N).$$

$$\mathbf{S} = \mathbf{R}_1^T \Delta \mathbf{R}_2$$

squeezing

basis change

**Bloch-Messiah reduction**



# Bloch-Messiah decomposition

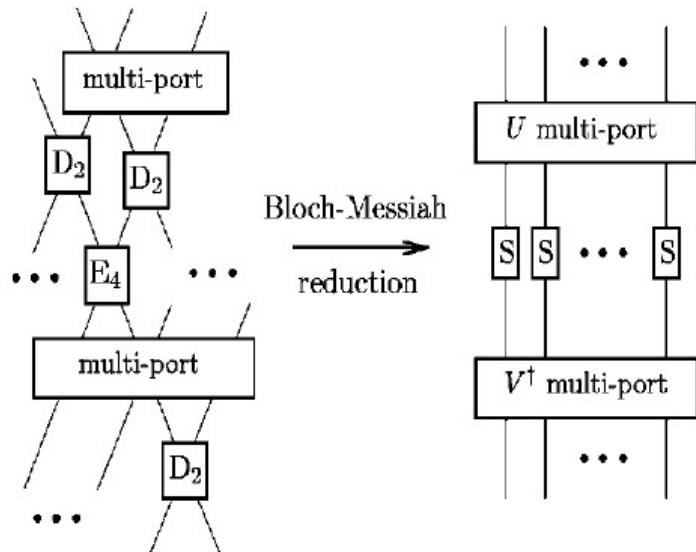
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## Squeezing as an irreducible resource

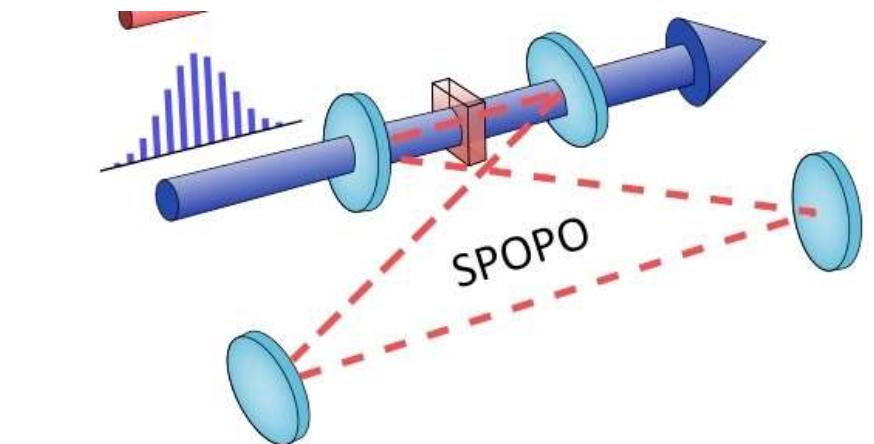
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$$\begin{pmatrix} \mathbf{q}_{out} \\ \mathbf{p}_{out} \end{pmatrix} = \mathbf{S} \begin{pmatrix} \mathbf{q}_{in} \\ \mathbf{p}_{in} \end{pmatrix}$$



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*collection of input modes*

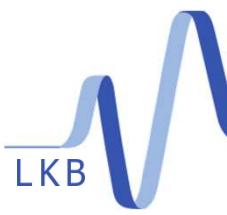
$$\{\mathbf{q}, \mathbf{p}\} = (q_1, q_2, \dots, q_N, p_1, p_2, \dots, p_N).$$

*input modes in vacuum -> discard R<sub>2</sub>*

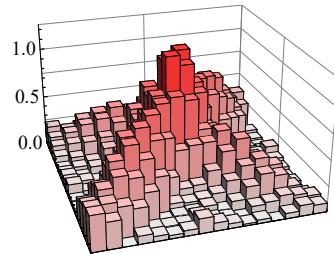
$$\mathbf{S} = \mathbf{R}_1^T \cancel{\Delta \mathbf{R}_2}$$

↑ squeezing  
↑ basis change

**Bloch-Messiah reduction**



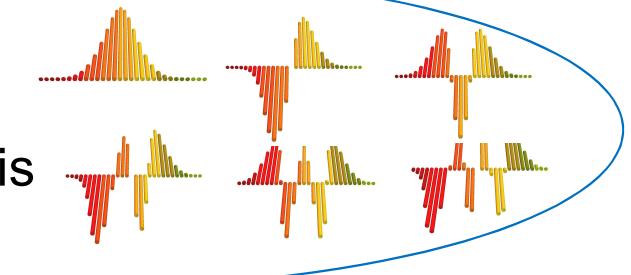
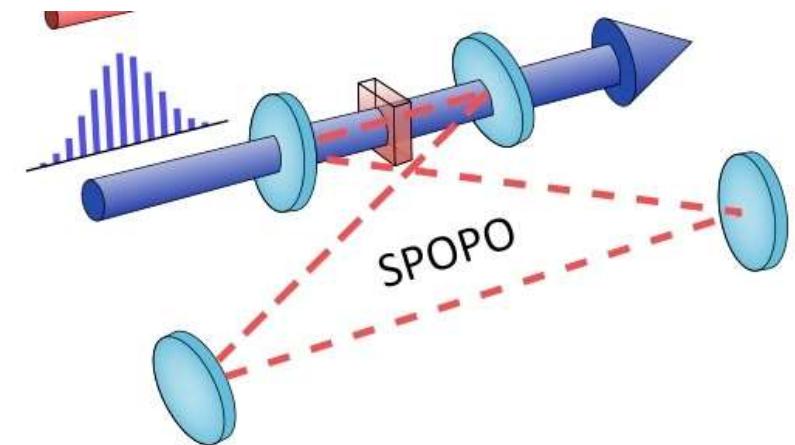
# Bloch-Messiah decomposition



multipartite entanglement  
in frequency-pixel basis

$$\begin{pmatrix} \mathbf{q}_{out} \\ \mathbf{p}_{out} \end{pmatrix} = R_1^T \Delta \begin{pmatrix} \mathbf{q}_{in} \\ \mathbf{p}_{in} \end{pmatrix}$$

squeezing in the  
supermodes basis

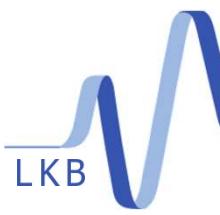


$$\begin{pmatrix} \mathbf{q}_{out} \\ \mathbf{p}_{out} \end{pmatrix} = S \begin{pmatrix} \mathbf{q}_{in} \\ \mathbf{p}_{in} \end{pmatrix}$$

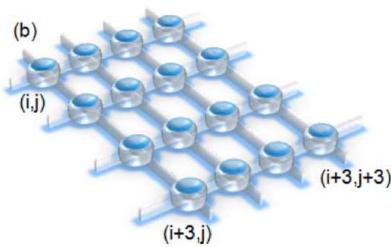
$$S = R_1^T \Delta R_2$$

squeezing  
basis change

**Bloch-Messiah reduction**



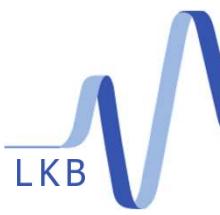
# Cluster states



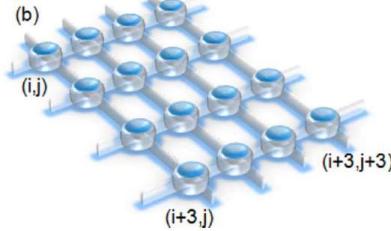
$$|\psi_V\rangle = \hat{C}_Z[V]|0\rangle_p^{\otimes N} = \prod_{j,k}^N e^{\frac{i}{2}V_{jk}\hat{q}_j\hat{q}_k}|0\rangle_p^{\otimes N} = e^{\frac{i}{2}\hat{q}^T V^i}|0\rangle_p^{\otimes N}$$

Collection of  $N$  infinitely p-squeezed states (modes)

Nullifiers  $\hat{p}_i - \sum_k V_{i,k} \hat{q}_k$        $\Delta^2 \left( \hat{p}_i - \sum_k V_{i,k} \hat{q}_k \right) = \langle \psi_V | \left( \hat{p}_i - \sum_k V_{i,k} \hat{q}_k \right)^2 | \psi_V \rangle \longrightarrow 0$



# Cluster states



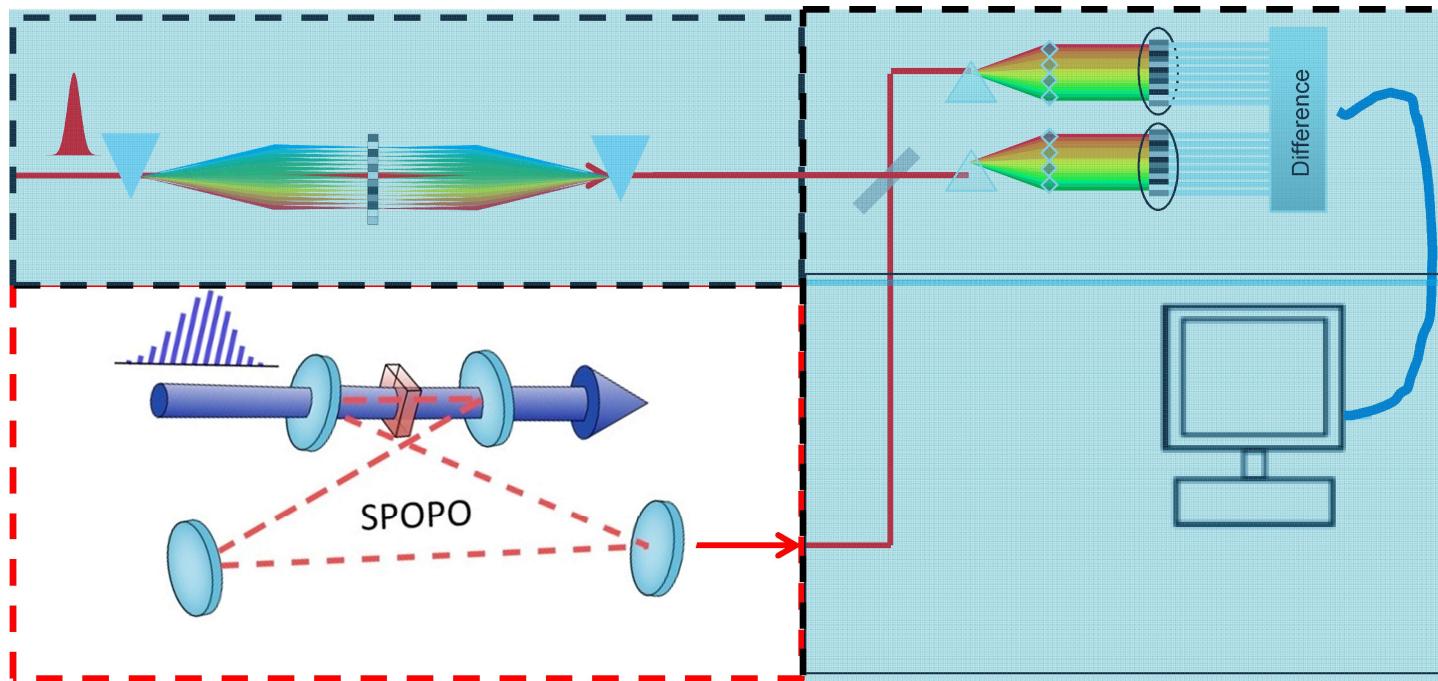
$$|\psi_V\rangle = \hat{C}_Z[V]|0\rangle_p^{\otimes N} = \prod_{j,k}^N e^{\frac{i}{2}V_{jk}\hat{q}_j\hat{q}_k}|0\rangle_p^{\otimes N} = e^{\frac{i}{2}\hat{q}^T V|i\rangle_p^{\otimes N}}$$

Collection of  $N$  infinitely p-squeezed states (modes)

multimode Gaussian state

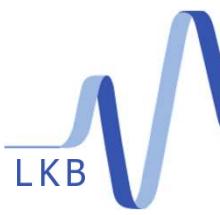
## Controlling B-M

$$\begin{pmatrix} q_{cluster} \\ p_{cluster} \end{pmatrix} = R_1^T \Delta \begin{pmatrix} q_{in} \\ p_{in} \end{pmatrix}$$

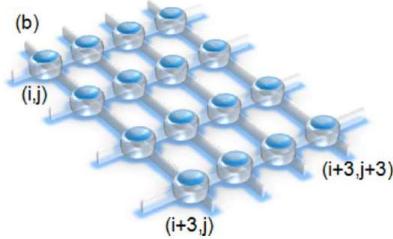


Control on  $R_1^T$   
(basis change)  
via mode-selective  
homodyne

G. Ferrini et al, New J Phys 15,  
093015 (2013)  
G. Ferrini et al., PRA 94,  
062332 (2016)



# Cluster states



$$|\psi_V\rangle = \hat{C}_Z[V]|0\rangle_p^{\otimes N} = \prod_{j,k}^N e^{\frac{i}{2}V_{jk}\hat{q}_j\hat{q}_k}|0\rangle_p^{\otimes N} = e^{\frac{i}{2}\hat{q}^T V|i\rangle_p^{\otimes N}}$$

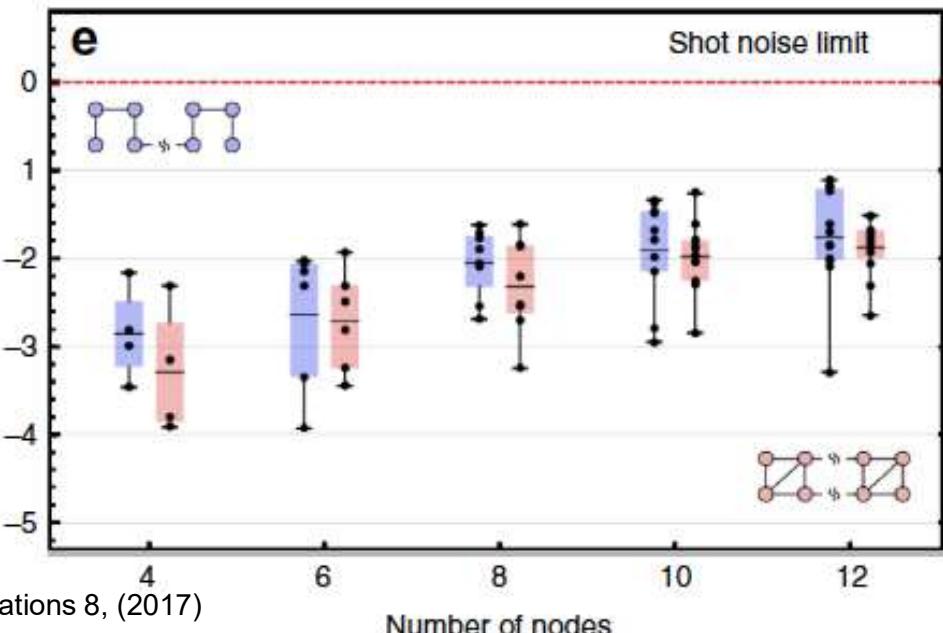
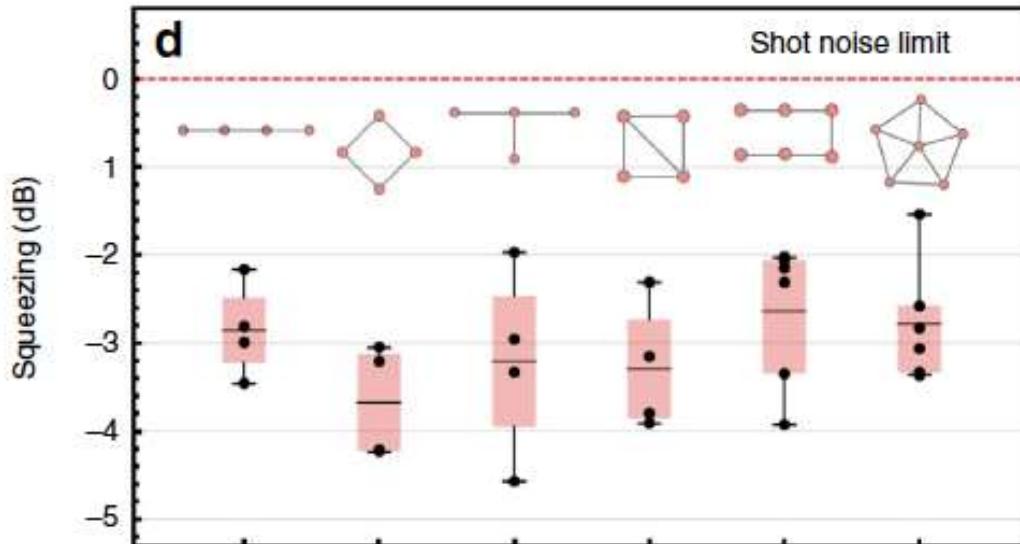
Collection of  $N$  infinitely p-squeezed states (modes)

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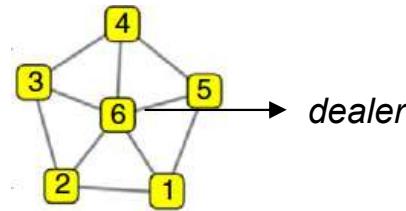
## Controlling B-M

$$\begin{pmatrix} q_{cluster} \\ p_{cluster} \end{pmatrix} = R_1^T \Delta \begin{pmatrix} q_{in} \\ p_{in} \end{pmatrix}$$

## Cluster states simulation in the current setup

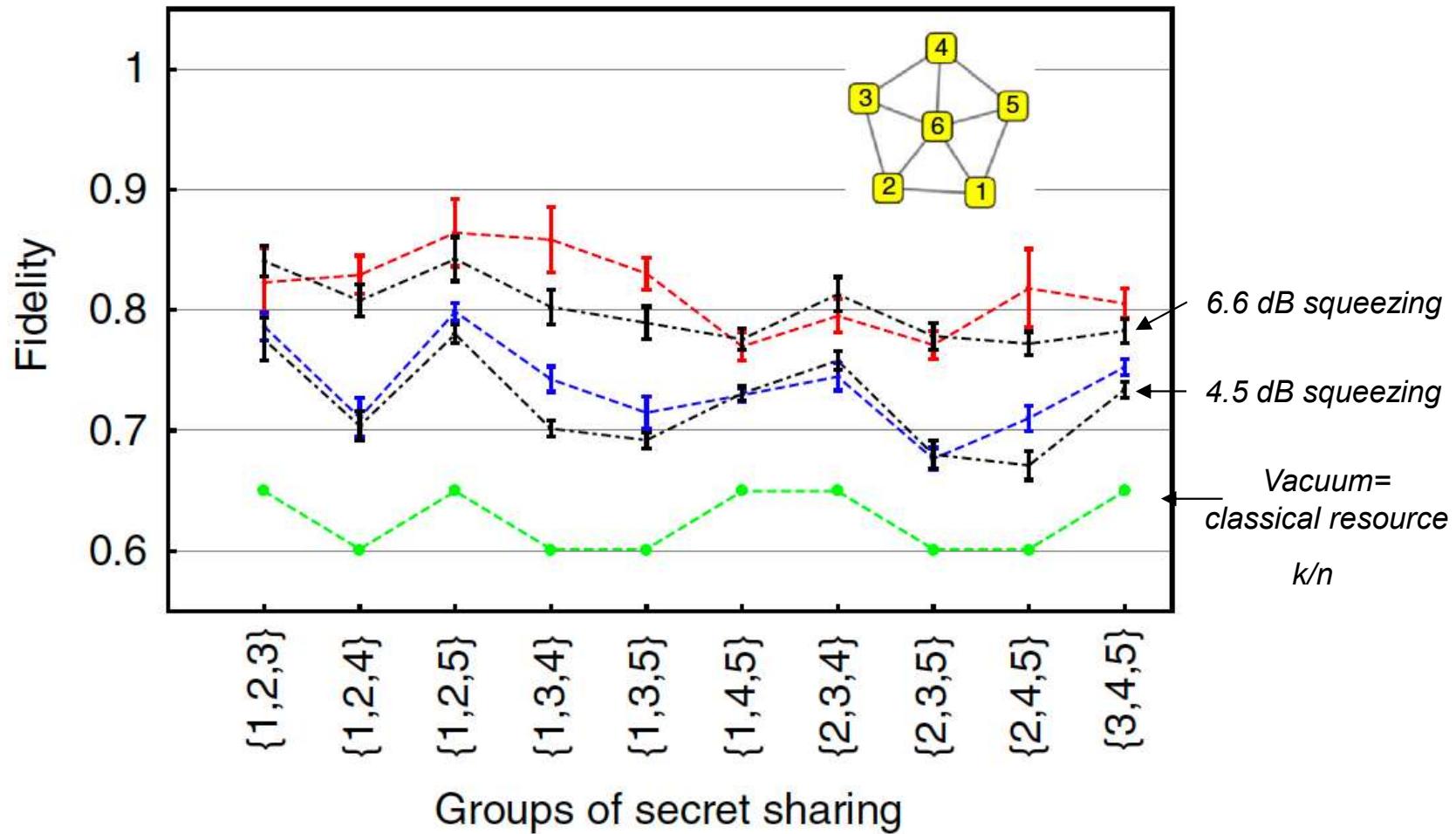


five-partite secret sharing protocol with six mode all-optical quantum graph



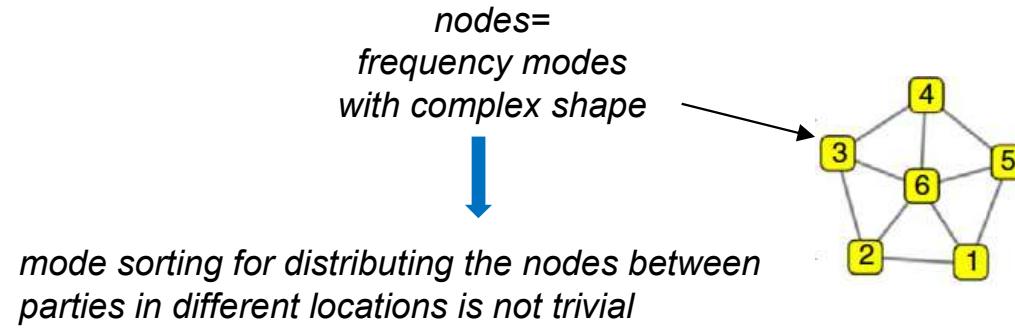
- ❖ Secret can only be retrieved through a collaboration of subsets of the 3 players
  
- ❖ quantum correlations increase both the protocol **security** as well as its retrieval **fidelity** compared to classical resources

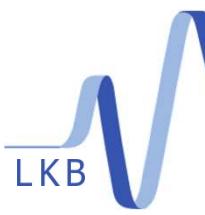
five-partite secret sharing protocol with six mode all-optical quantum graph



# A cluster for secret sharing

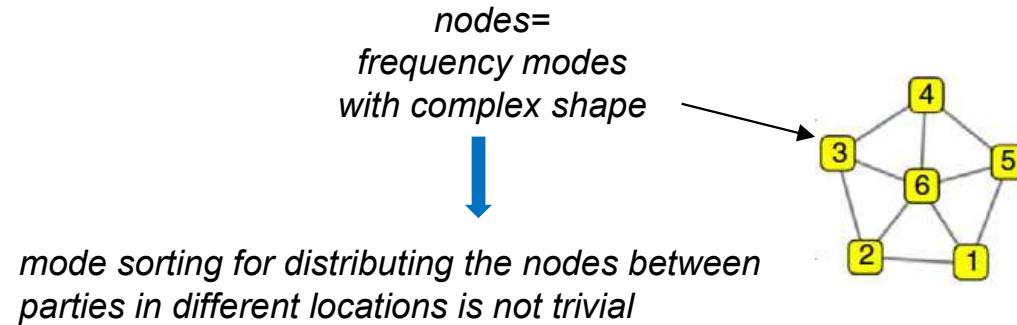
five-partite secret sharing protocol with six mode all-optical quantum graph



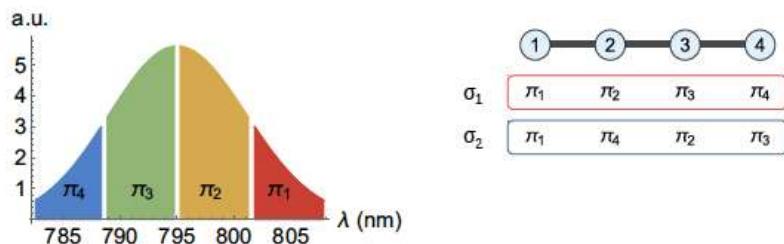


# A cluster for secret sharing

five-partite secret sharing protocol with six mode all-optical quantum graph

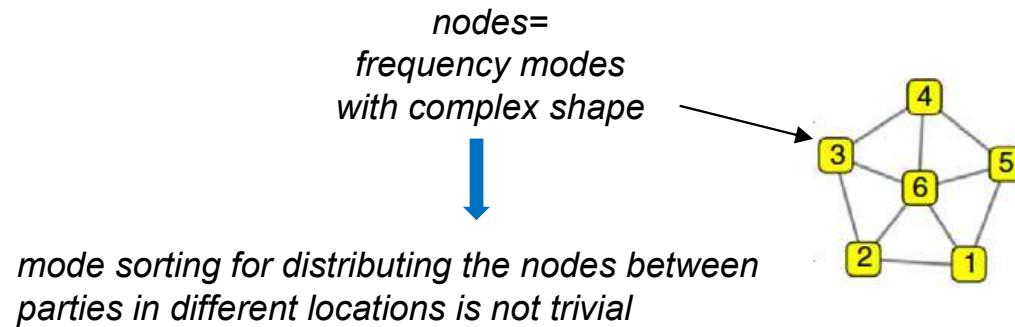


can we have nodes= frequency bands?

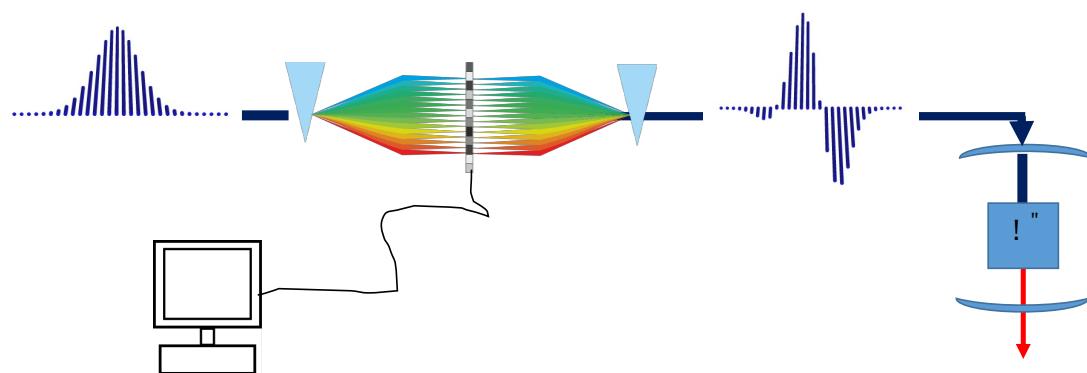


# A cluster for secret sharing

five-partite secret sharing protocol with six mode all-optical quantum graph



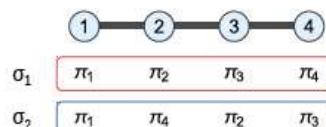
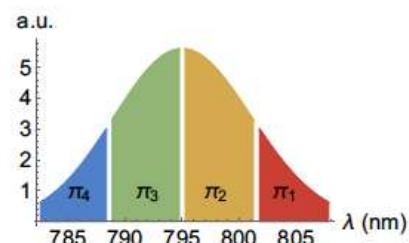
can we have nodes= frequency bands?



*pump shaping*

optimization procedure based on an evolutionary algorithm

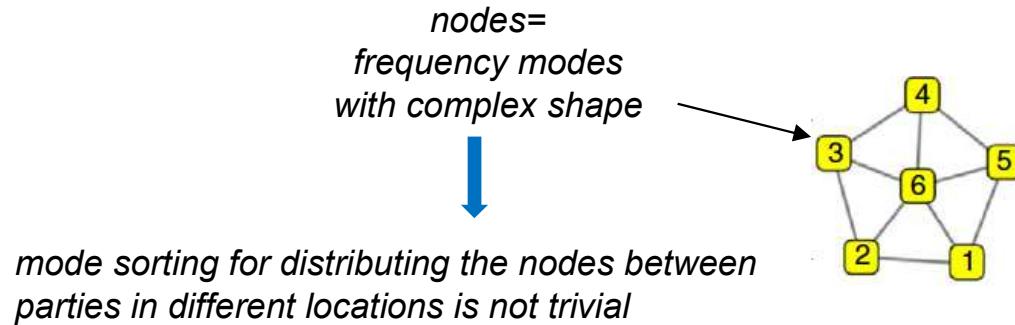
F. Arzani et al, arXiv:1709.10055





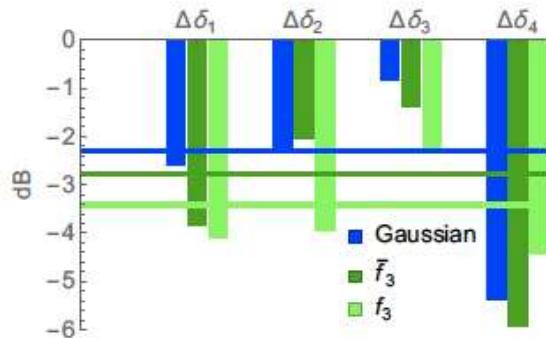
# A cluster for secret sharing

five-partite secret sharing protocol with six mode all-optical quantum graph

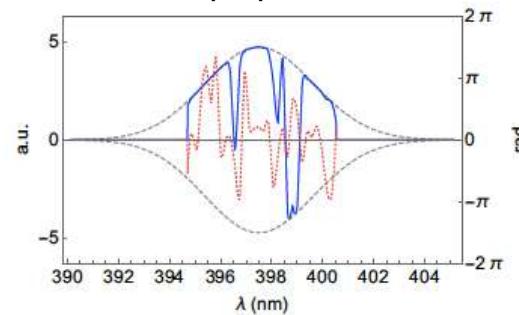


can we have nodes= frequency bands?

Nullifiers



Pump spectrum

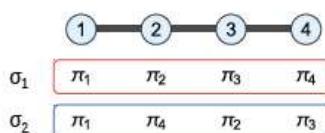
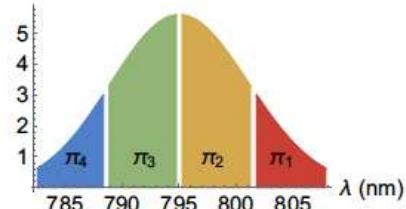


pump shaping

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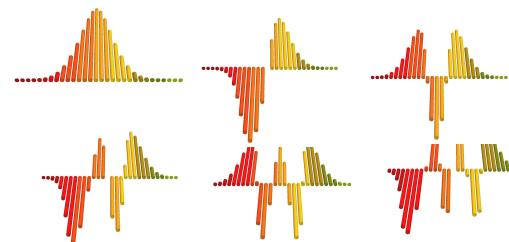
F. Arzani et al, arXiv:1709.10055

a.u.



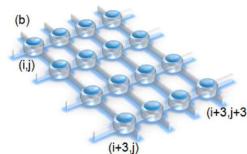
## Multimode quantum optics

multipartite entanglement

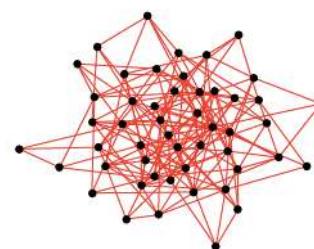


## Study of quantum network

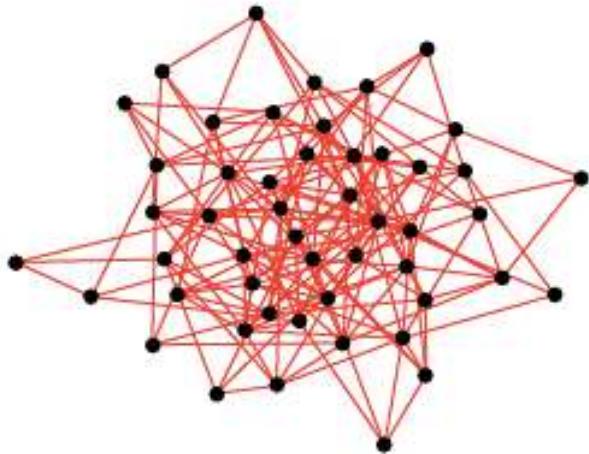
cluster states



quantum complex networks



Collections of quantum systems arranged in a non-regular topology

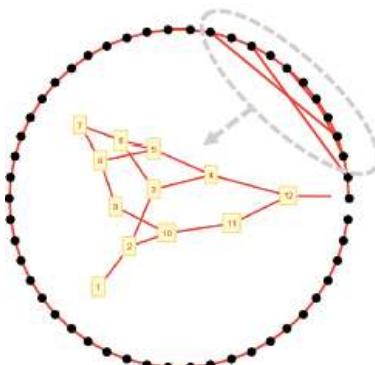


*Ex: random-network*

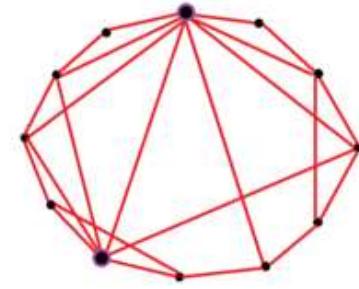
Collections of quantum systems arranged in a non-regular topology



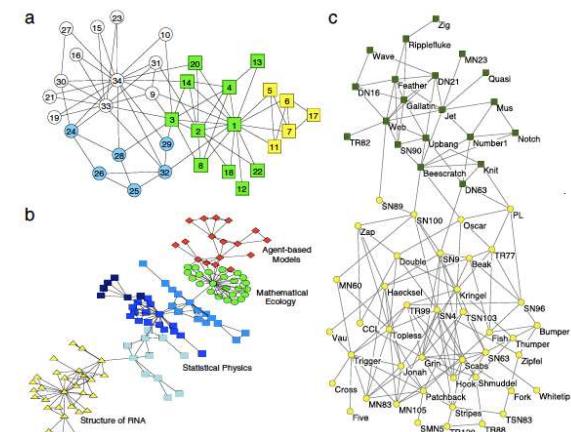
*Ex: random-network*



*linear chain with shortcuts*



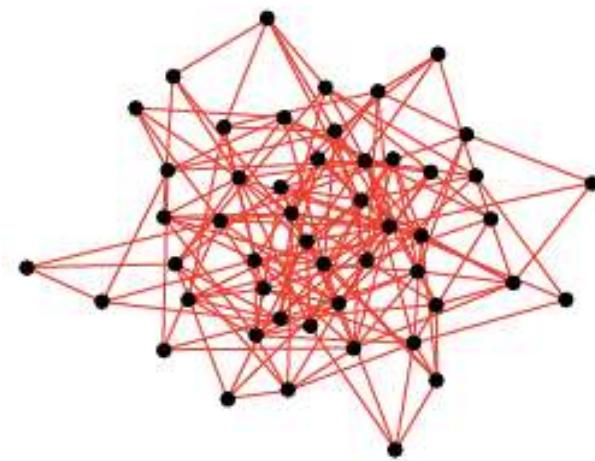
*small-world network*



*community structures*

Collections of quantum systems arranged in a non-regular topology

Why interesting ?



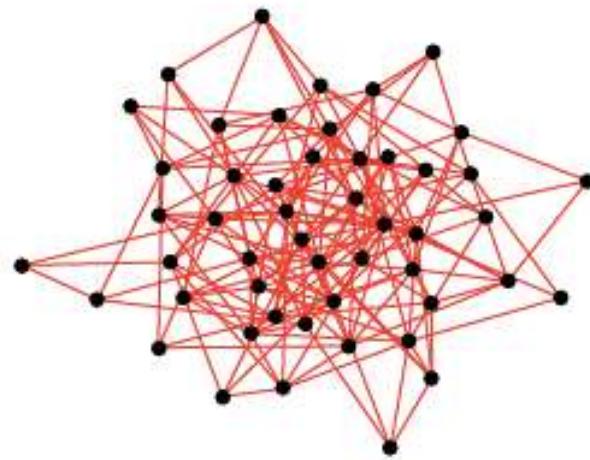
**1) quantum description of complex network** = different from classical case  
localization, quantum walk, phase transition, quantum transport, synchronization, etc

**2) quantum networks with no classical equivalent ( ex. entanglement connections)**  
= Future quantum communication and information technologies -> ( computational complexity,  
but also complex topology)

**3) quantum complex networks useful in several contexts,**  
e.g. open system dynamics, quantum gravity

Collections of quantum systems arranged in a non-regular topology

Why interesting ?



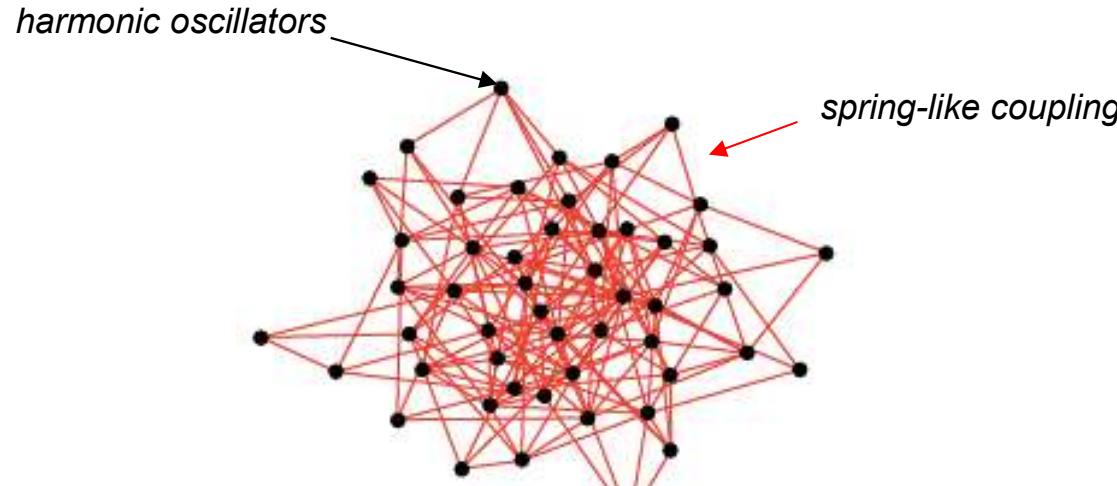
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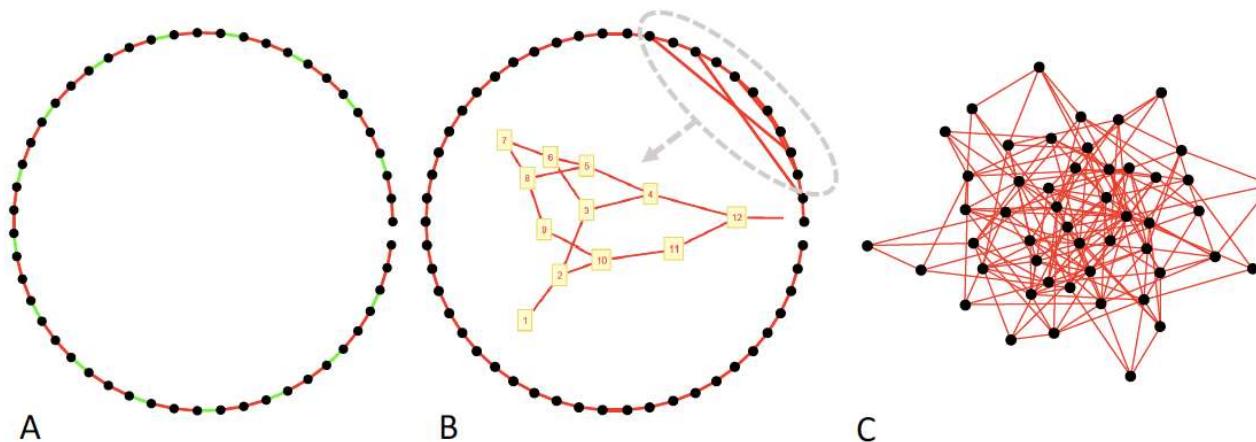
**3) quantum complex networks useful in several contexts,**  
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## Our proposal: reconfigurable optical implementation

Collections of quantum systems arranged in a non-regular topology



reconfigurable coupling and topology,  
relevant size (here 50 nodes)



## Network dynamics

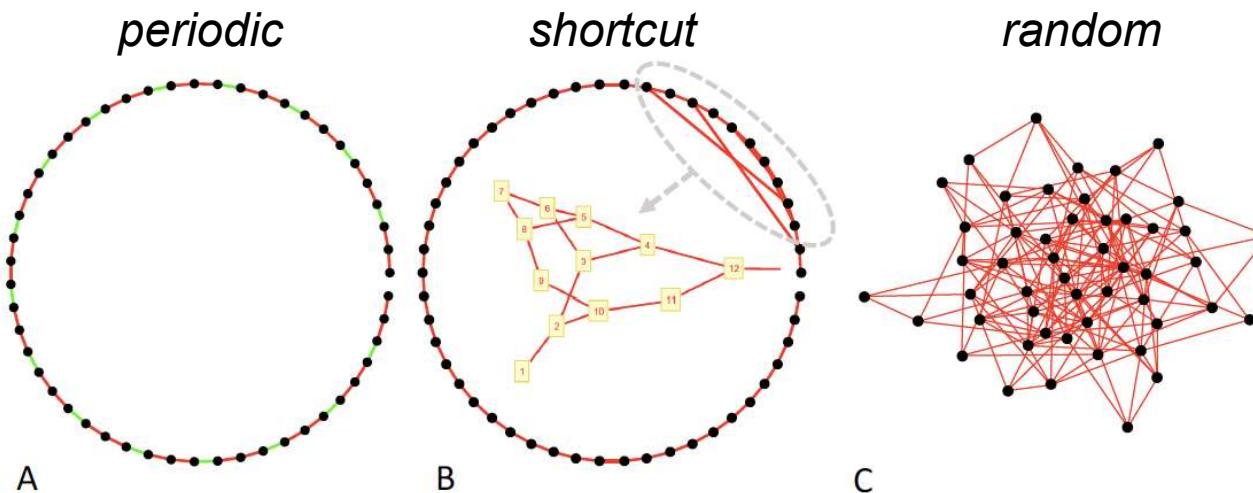
$$H_E = \frac{\mathbf{p}^T \Delta_\omega \mathbf{p}}{2} + \mathbf{q}^T \sqrt{\Delta_\omega^{-1}} \mathbf{A} \sqrt{\Delta_\omega^{-1}} \mathbf{q}$$

$$\mathbf{A}_{ii} = \omega_i^2/2 + \sum_j g_{ij}/2$$

$$\mathbf{A}_{i \neq j} = g_{ij}/2$$

$$\Delta_\omega = \text{diag}\{\omega_1, \dots, \omega_N\}$$

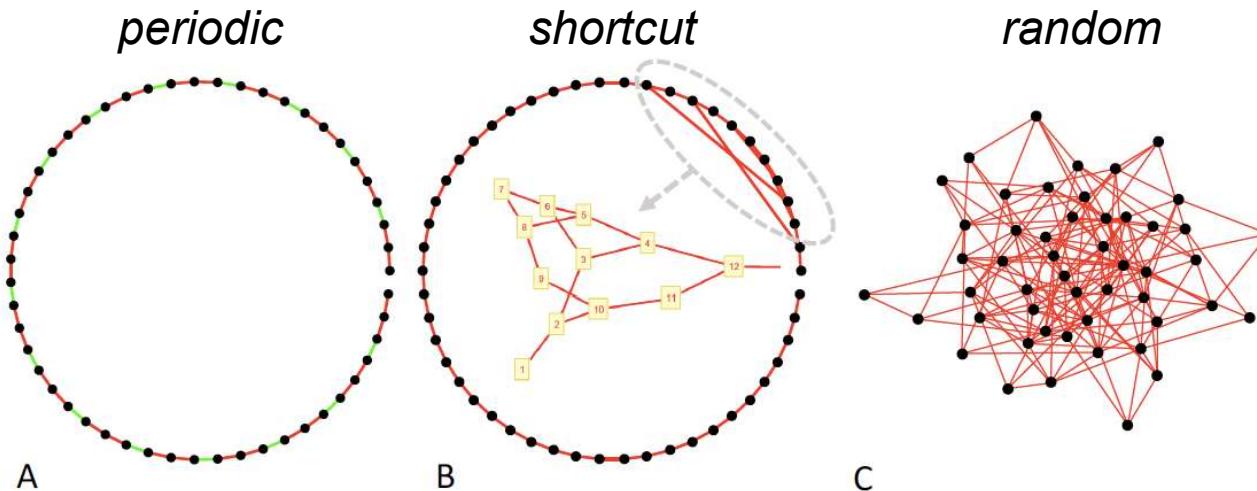
$$\begin{aligned} \mathbf{q}^T &= (q_1, q_2, \dots, q_N). \\ \mathbf{p}^T &= (p_1, p_2, \dots, p_N) \end{aligned}$$



## Network dynamics

$$H_E = \frac{\mathbf{p}^T \Delta_\omega \mathbf{p}}{2} + \mathbf{q}^T \sqrt{\Delta_\omega^{-1}} \mathbf{A} \sqrt{\Delta_\omega^{-1}} \mathbf{q}$$

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^\Omega T_1^{-1} & T_1 D_{\sin}^\Omega T_2^{-1} \\ -T_2 D_{\sin}^\Omega T_1^{-1} & T_2 D_{\cos}^\Omega T_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$

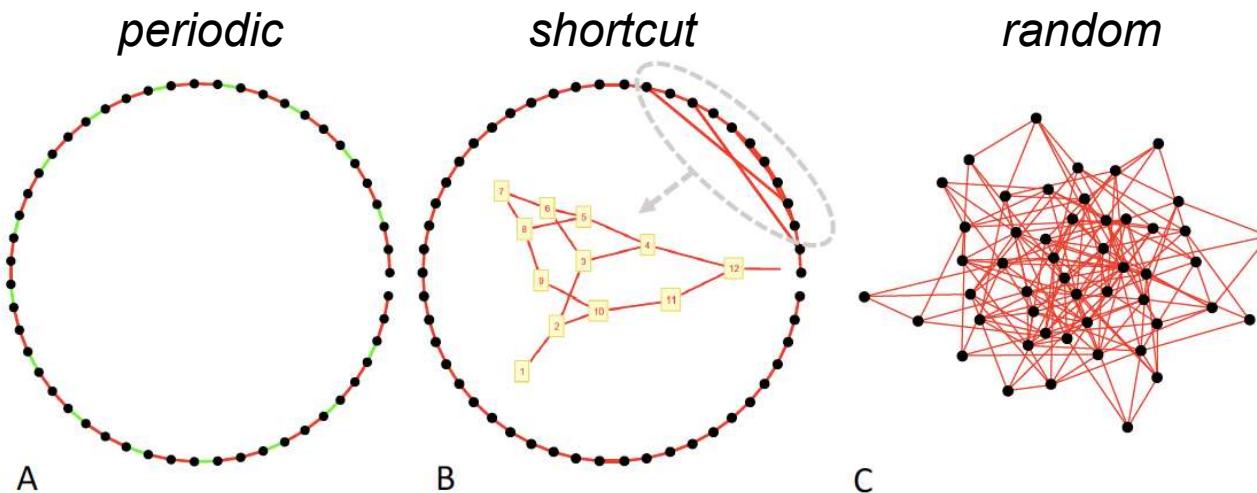


## Network dynamics

$$H_E = \frac{\mathbf{p}^T \Delta_\omega \mathbf{p}}{2} + \mathbf{q}^T \sqrt{\Delta_\omega^{-1}} \mathbf{A} \sqrt{\Delta_\omega^{-1}} \mathbf{q}$$

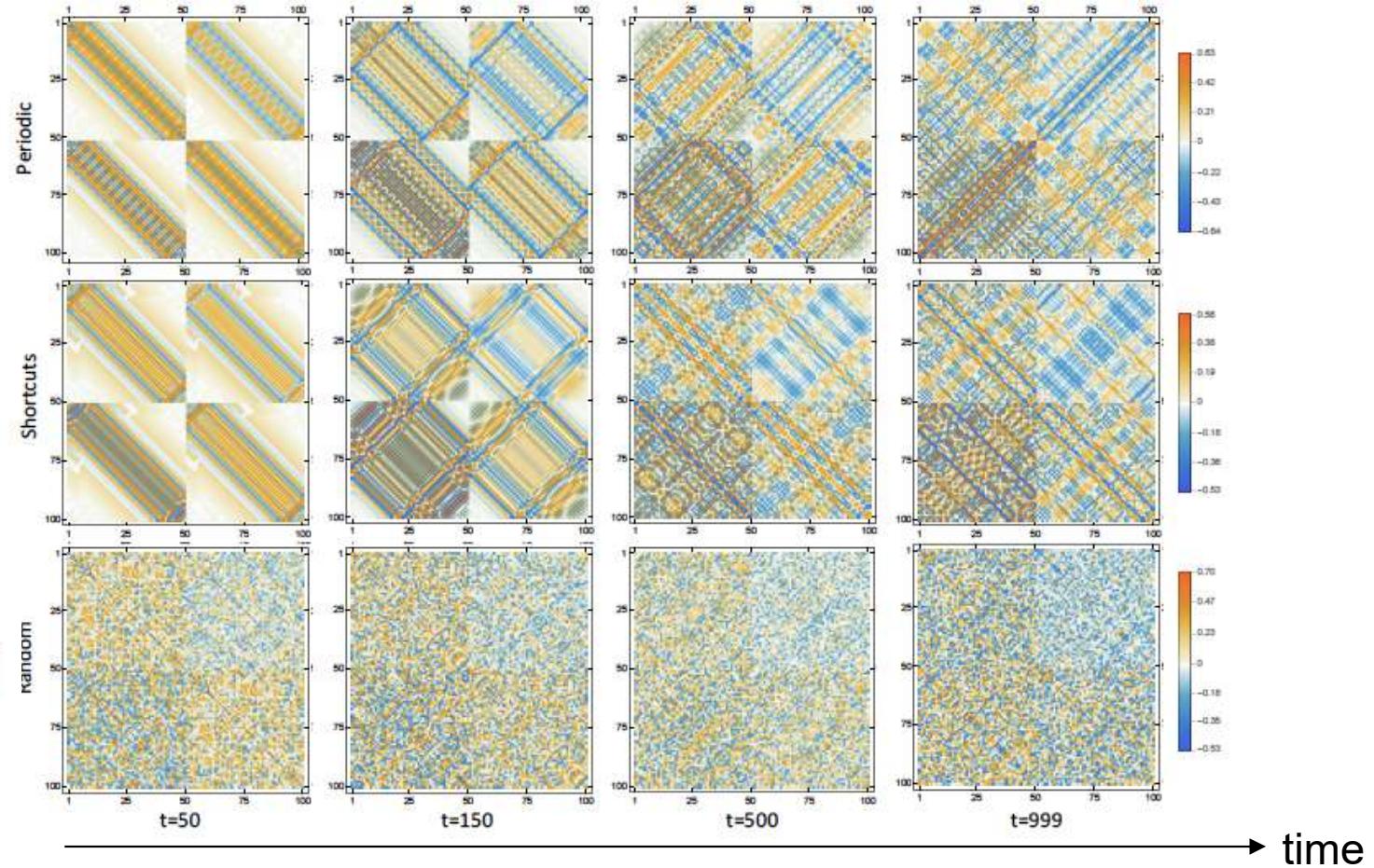
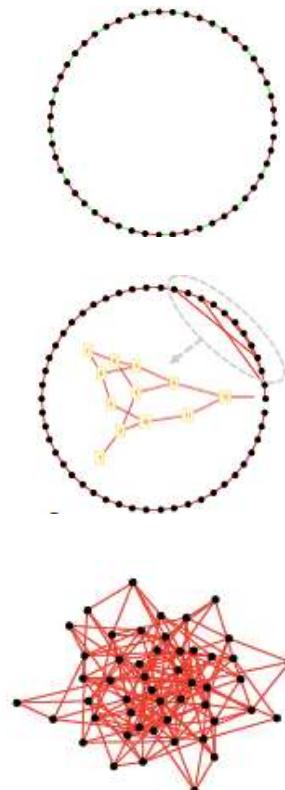
$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^\Omega T_1^{-1} & T_1 D_{\sin}^\Omega T_2^{-1} \\ -T_2 D_{\sin}^\Omega T_2^{-1} & T_2 D_{\cos}^\Omega T_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$

symplectic transformations



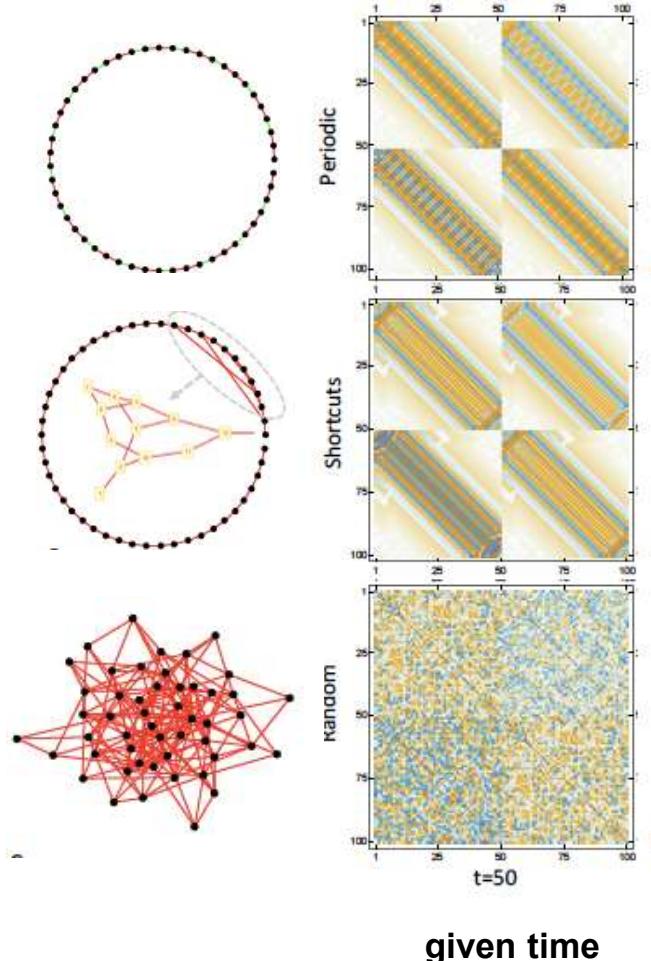
## Network dynamics

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & S \mathbf{V} \begin{pmatrix} T_1 D_{\sin}^{\Omega} T_2^{-1} \\ T_2 D_{\cos}^{\Omega} T_2^{-1} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$



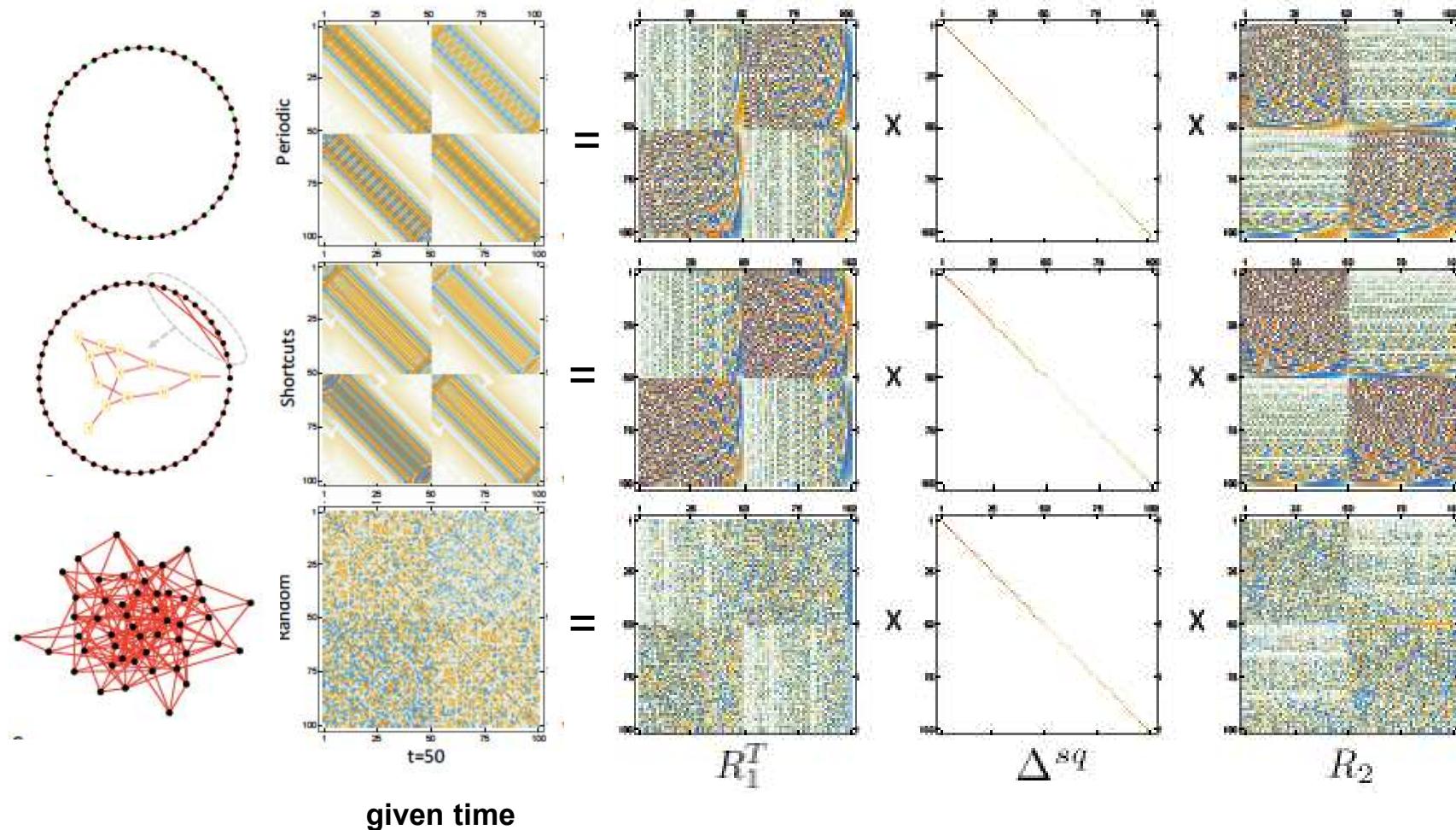
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$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & T_2 D_{\sin}^{\Omega} T_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$



## Network dynamics -> Bloch-Messiah reduction

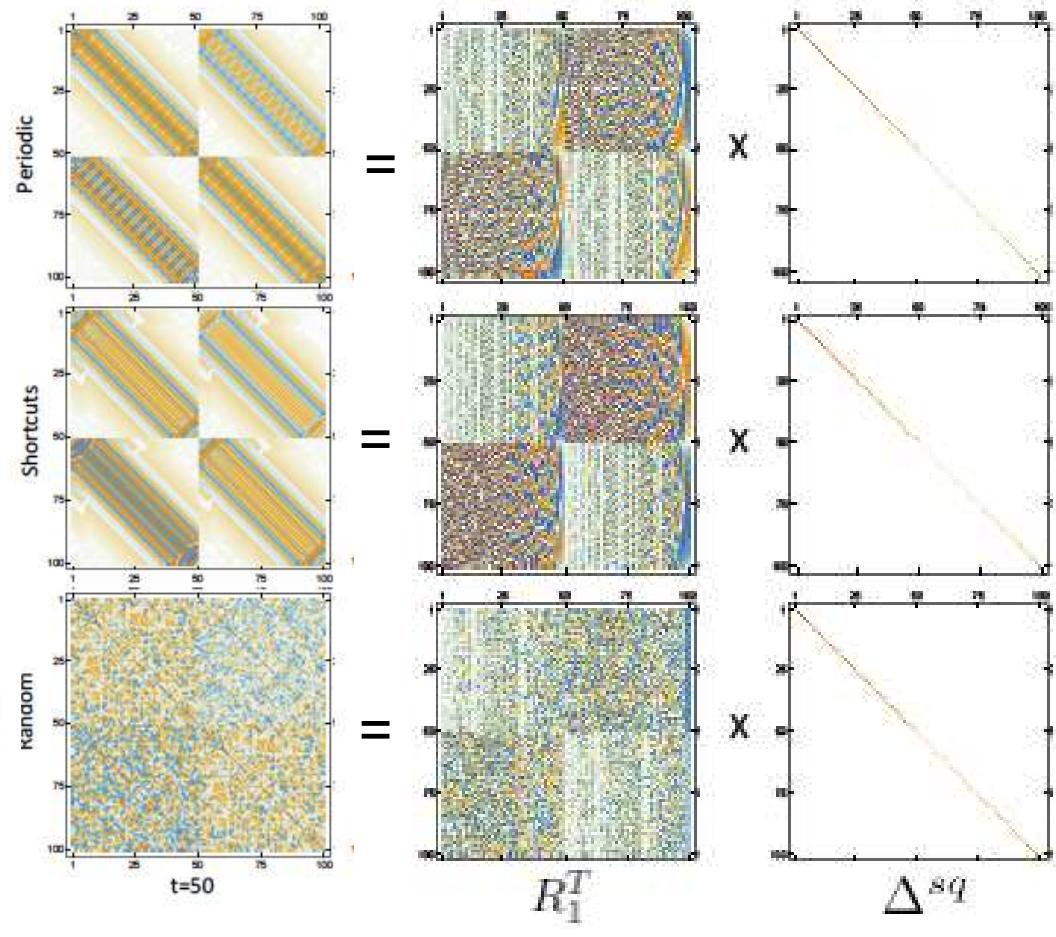
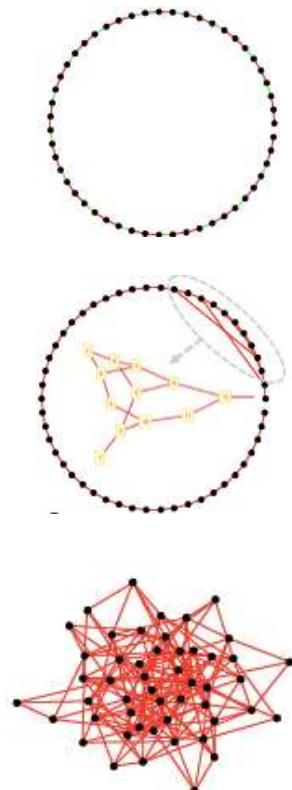
$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & S \\ -T_2 D_{\sin}^{\Omega} & S \mathbf{V}^{-1} T_2 D_{\sin}^{\Omega} T_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix} = R_1^T \Delta^{sq} R_2 \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$



## Network dynamics -> Bloch-Messiah reduction

if vacuum

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & S \mathbf{V} \begin{pmatrix} T_1 D_{\sin}^{\Omega} T_2^{-1} \\ T_2 D_{\cos}^{\Omega} T_2^{-1} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix} = R_1^T \Delta^{sq} R_2 \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$



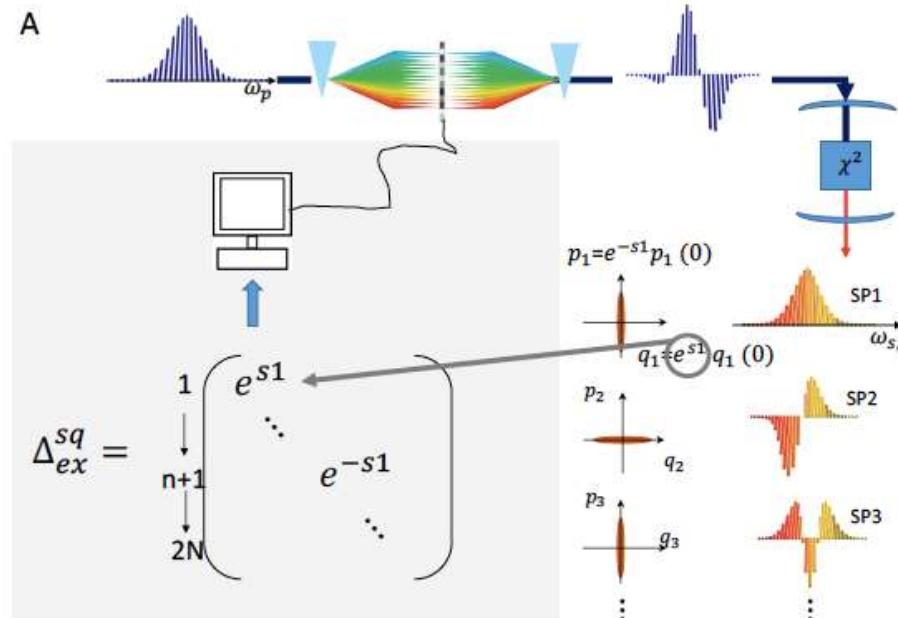
given time



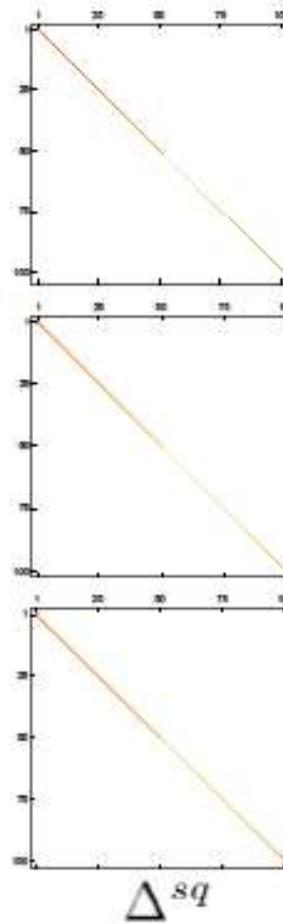
# Quantum complex networks

Optical implementation-> Bloch-Messiah reduction  
*if vacuum*

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & T_2 D_{\sin}^{\Omega} T_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix} = R_1^T \Delta^{sq} R_2 \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$



given time



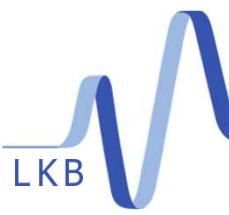
experimental control

$\Delta_{ex}^{sq}$

pump shaping

optimization procedure  
based on an  
evolutionary algorithm

F. Arzani et al, arXiv:1709.10055

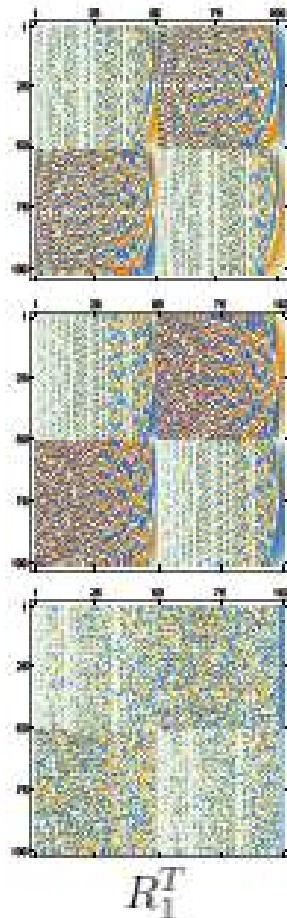
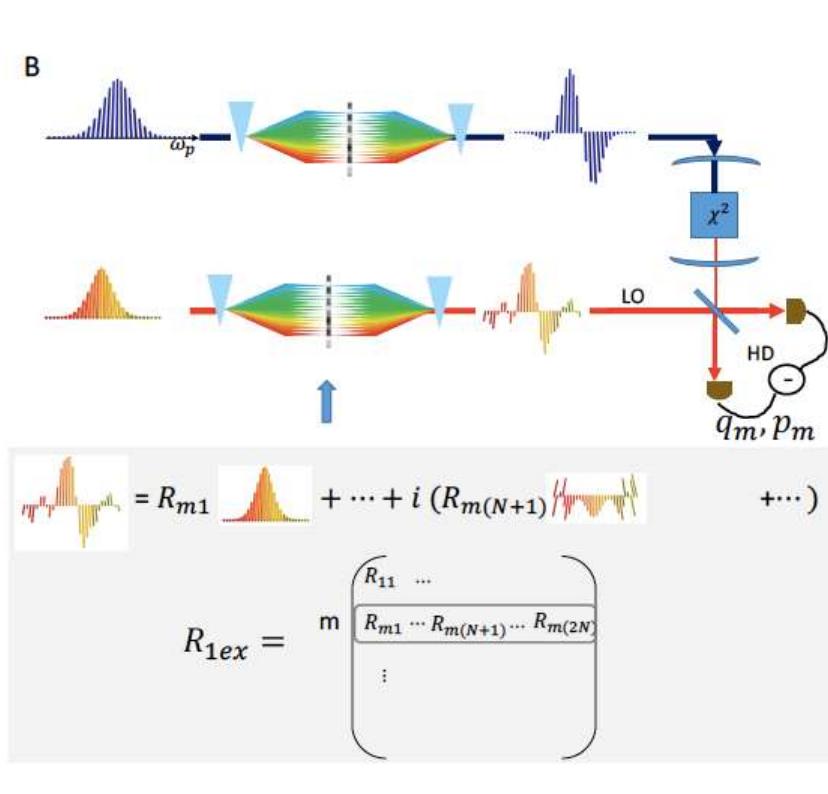


# Quantum complex networks

Network dynamics -> Bloch-Messiah reduction

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & S \mathbf{V} \begin{pmatrix} T_1 D_{\sin}^{\Omega} T_2^{-1} \\ T_2 D_{\cos}^{\Omega} T_2^{-1} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix} = R_1^T \Delta^{sq} R_2 \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$

if vacuum



given time

experimental control

$R_{1ex}$

choosing the measurement basis

$m$ -th nodes encoded in the mode defined by the  $m$ -th row of  $R1$



# Quantum complex networks

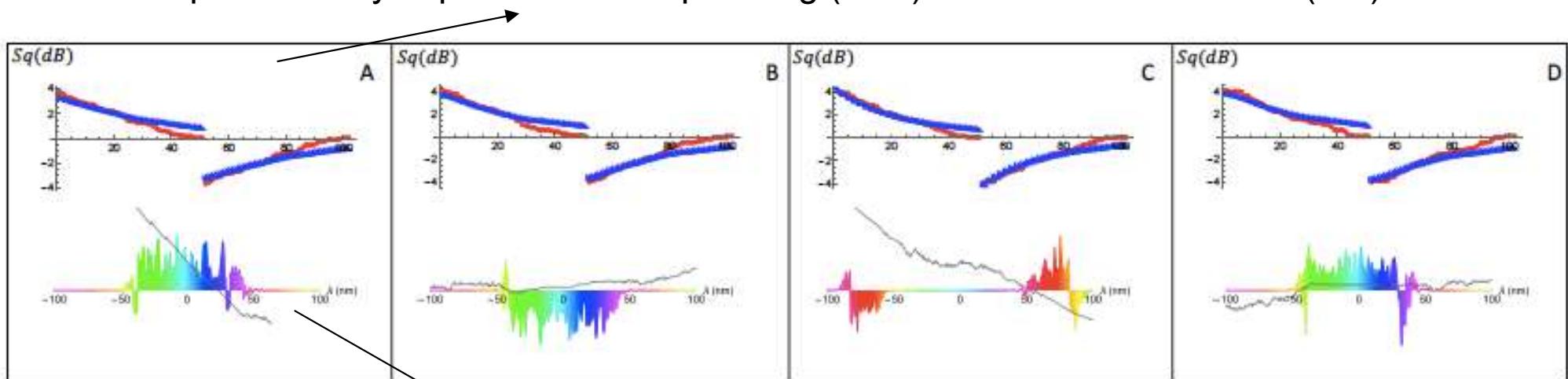
Network dynamics -> Bloch-Messiah reduction

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & T_2 D_{\sin}^{\Omega} T_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix} = R_1^T \Delta^{sq} R_2 \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$

if vacuum

experimental control

experimentally implementable squeezing (blue) and theoretical values (red)



time



shape of the mode encoding the 26-th oscillators

BiBO 2.5 mm



# Quantum complex networks

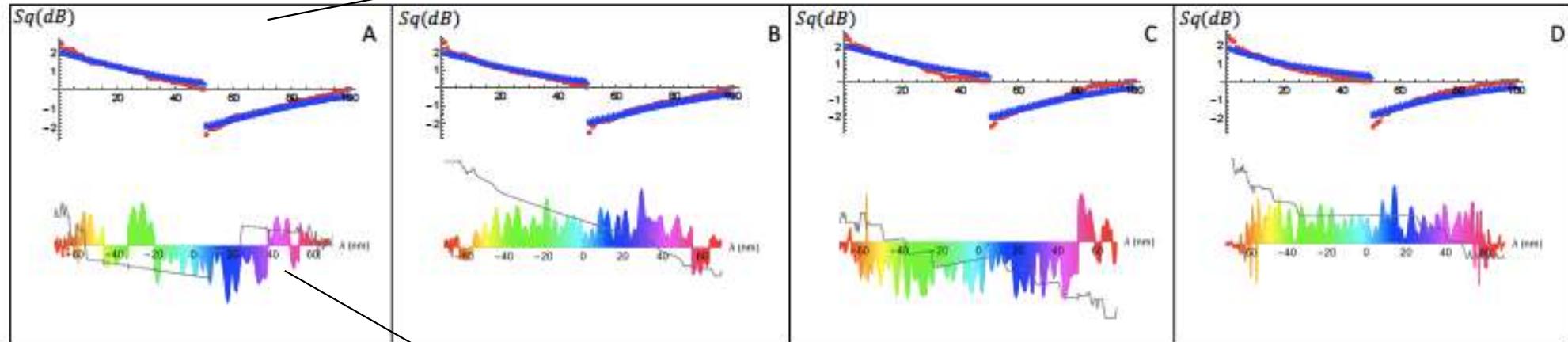
Network dynamics -> Bloch-Messiah reduction

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & S \mathbf{V} \begin{pmatrix} T_1 D_{\sin}^{\Omega} T_2^{-1} \\ T_2 D_{\cos}^{\Omega} T_2^{-1} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix} = R_1^T \Delta^{sq} R_2 \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$

if vacuum

experimental control

experimentally implementable squeezing (blue) and theoretical values (red)



shape of the mode encoding the 26-th oscillators

*BiBO 1.5 mm*



# Quantum complex networks

Network dynamics -> Bloch-Messiah reduction

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & T_2 D_{\sin}^{\Omega} T_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix} = R_1^T \Delta^{sq} R_2 \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$

**experimental control**

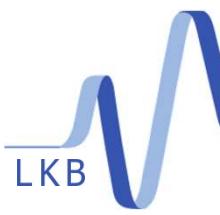
experimentally implementable squeezing (blue) and theoretical values (red)

if not vacuum

Initialization from vacuum can be included in the dynamics if state is pure and Gaussian

or

initial larger (2N) multimode entangled state -> tracing over half mode to obtain a mixed (thermal) state



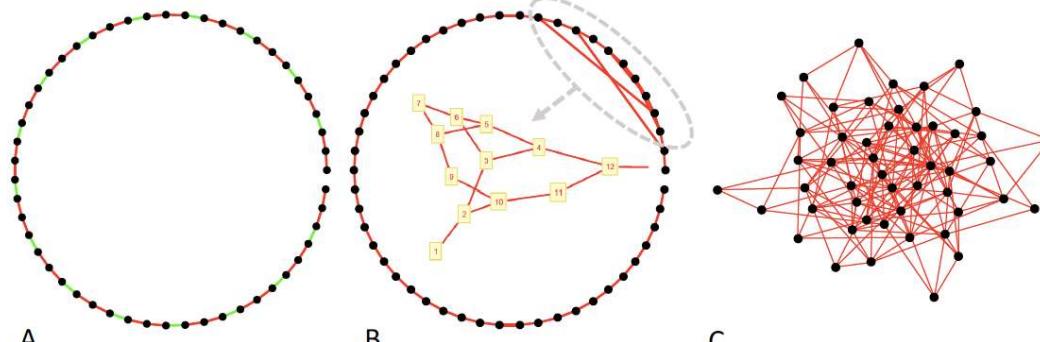
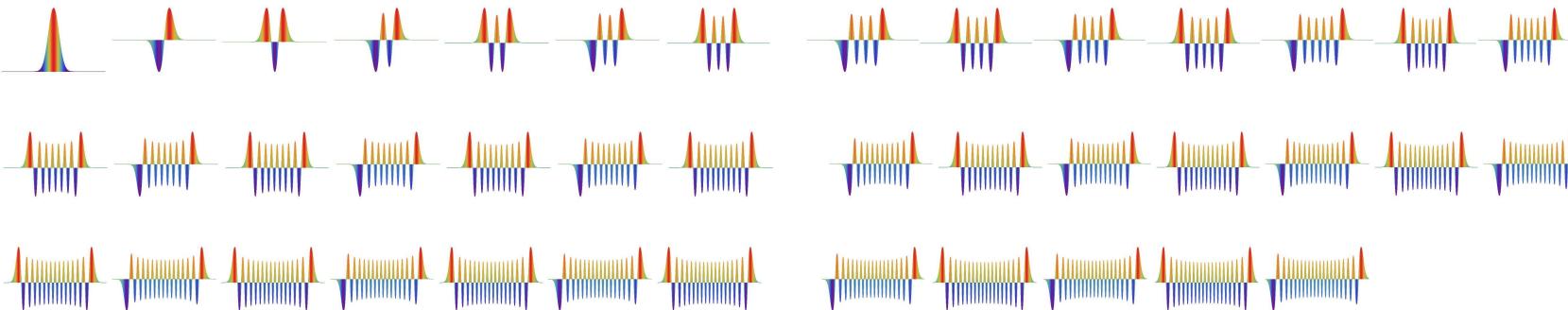
# Quantum complex networks

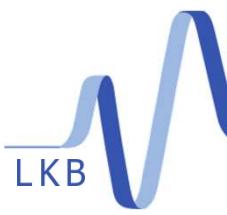
## Experimental feasibility

At least 50 nodes needed

Current setup

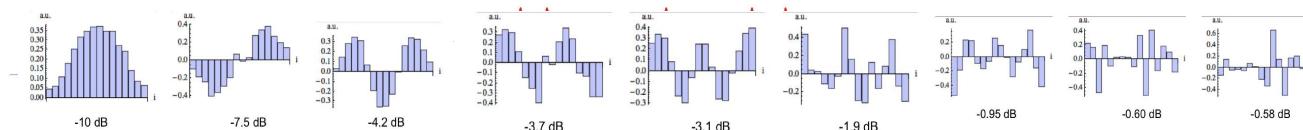
50 Hermite-Gauss modes with quantum properties are generated (we can go up to 100)





# Quantum complex networks

*At least 50 nodes needed*

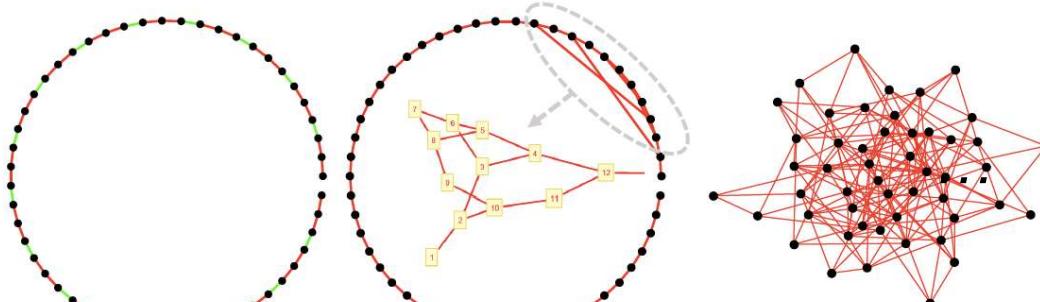


**Current setup**

**50 Hermite-Gauss modes with quantum properties are generated (we can go up to 100)**

*Detection of the first 16 has been done*

↓  
*Work in progress on coherent broadening of the LO spectrum*



## Non-Gaussian operations : coherent mode dependent single-photon subtraction

Signal

1    2    3

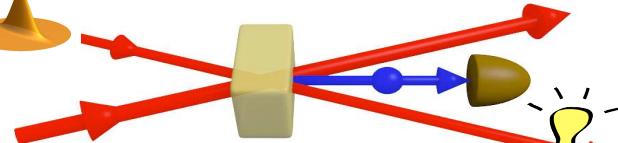
Gate

$|\alpha_1, \alpha_2, \dots\rangle$

Young-Sik Ra, Clément Jacquard, Adrien Dufour, Claude Fabre,  
and Nicolas Treps Phys. Rev. X 7, 031012 (2017)

M. Walschaers, C. Fabre, V. Parigi, N. Treps, accepted in Phys. Rev.Lett

$$\propto (\alpha_1 \hat{a}_1 + \alpha_2 \hat{a}_2 + \dots) |\psi_s\rangle$$



## Non-Gaussian operations : coherent mode dependent single-photon subtraction

Signal

1 2 3

Gate

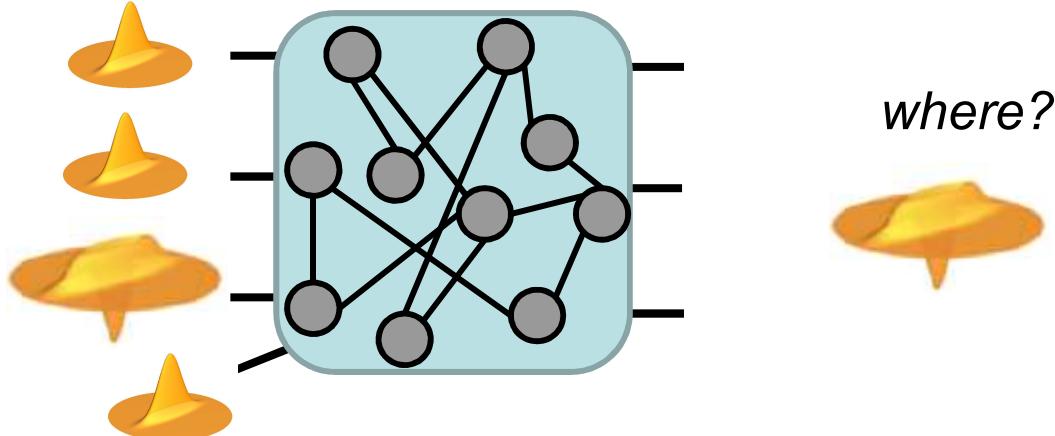
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$$\propto (\alpha_1 \hat{a}_1 + \alpha_2 \hat{a}_2 + \dots) |\psi_s\rangle$$

### ❖ Transport of non-Gaussian information in complex networks



## Non-Gaussian operations : coherent mode dependent single-photon subtraction

Signal

1 2 3

Gate

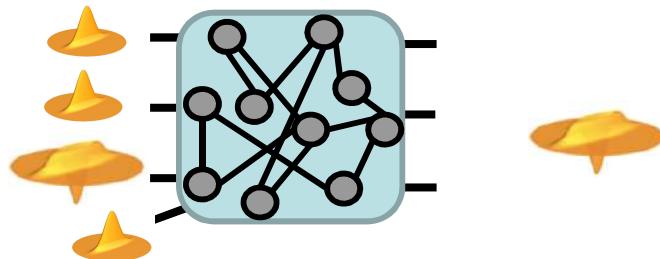
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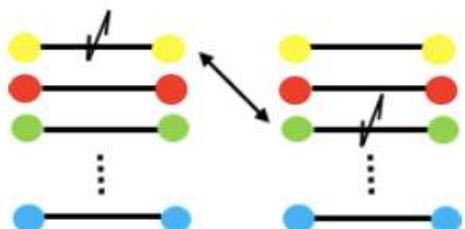
$$\propto (\alpha_1 \hat{a}_1 + \alpha_2 \hat{a}_2 + \dots) |\psi_s\rangle$$

### ❖ Transport of non-Gaussian information in complex networks



### ❖ All optical quantum repeaters for quantum communication

collaboration with Peter van Loock



optimal nonlocal preparation of approximate DV Bell states  
via photon- subtraction + optimal entanglement swapping  
via multiplexing

probability of the process scale as  $(1 - P)^N$ ,  $N = \text{number pf modes}$

# *Thank you!*



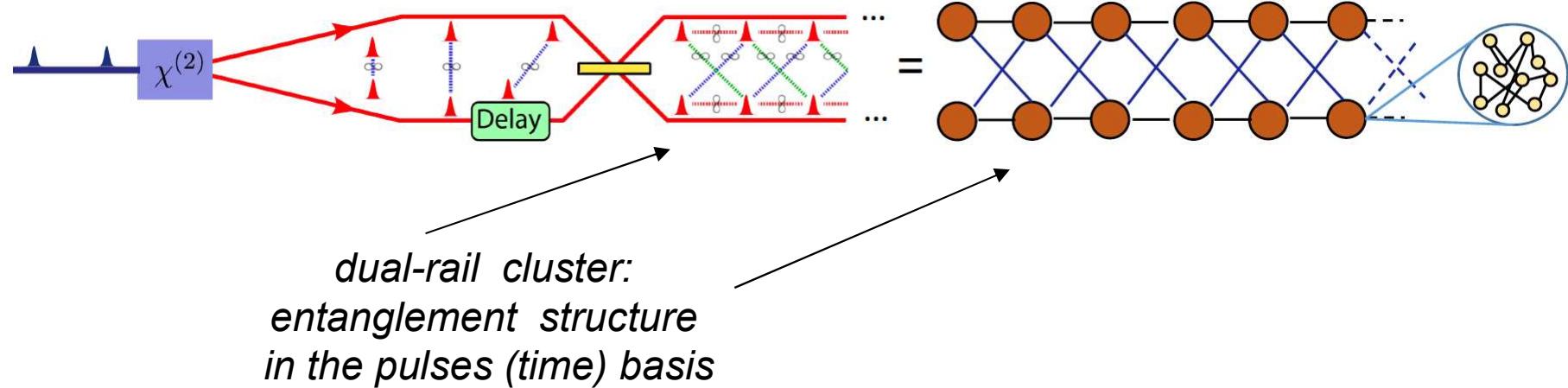




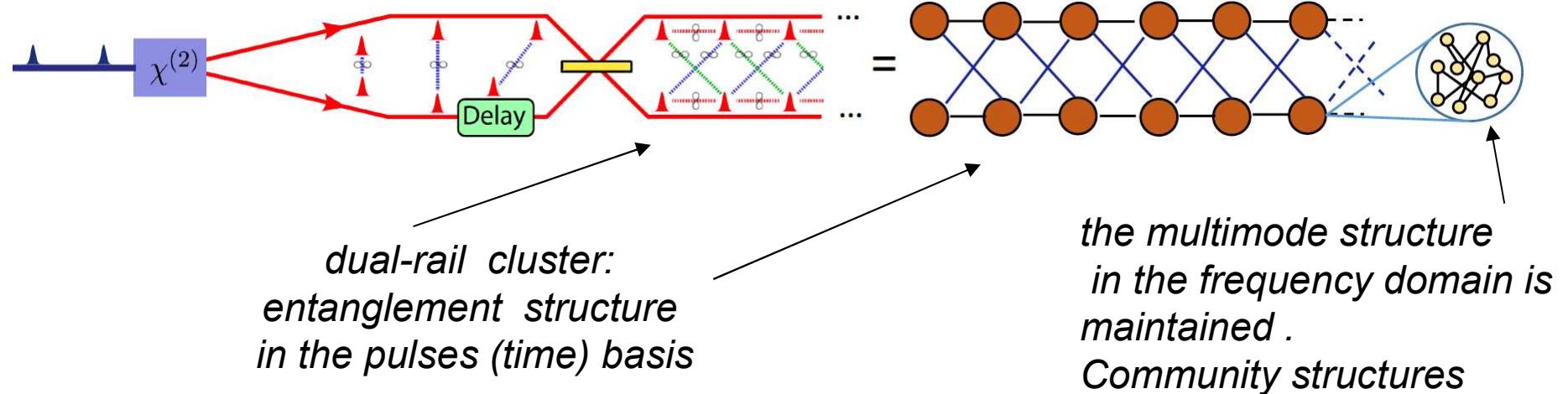


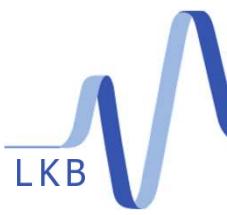


- **Number of modes : 16** at the moment but they are only limited by the measurement procedure -> they could be around **40** if we increase the spectral bandwidth of LO
- Exploring new experimental setup -> “big-states”  **$10^5$  modes**



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- Exploring new experimental setup -> “big-states”  **$10^5$  modes**





# Quantum complex networks

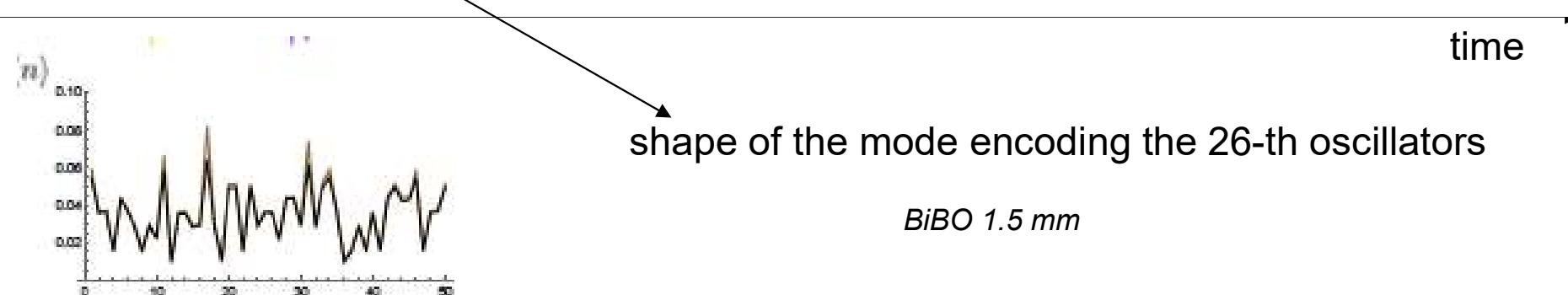
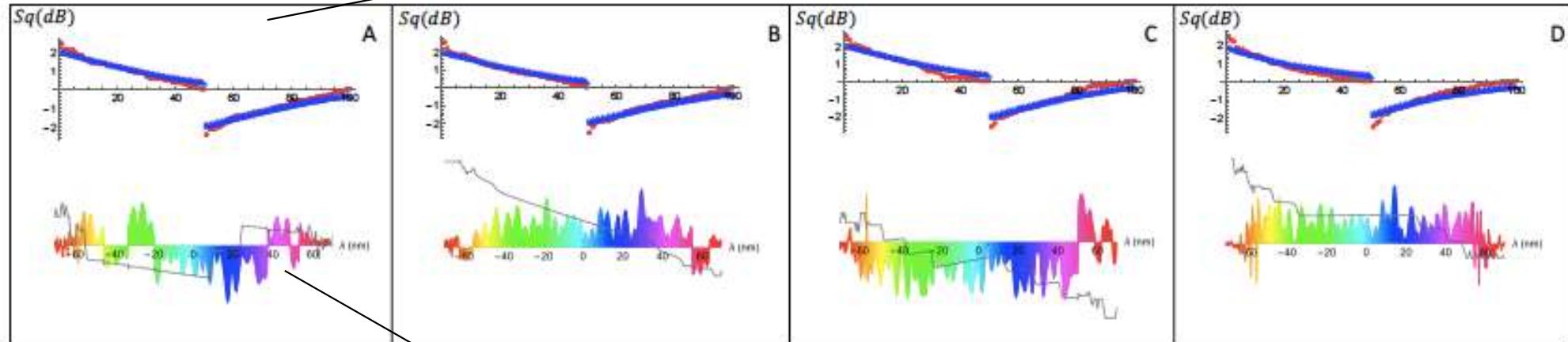
Network dynamics -> Bloch-Messiah reduction

$$\begin{pmatrix} \mathbf{q}(t) \\ \mathbf{p}(t) \end{pmatrix} = \begin{pmatrix} T_1 D_{\cos}^{\Omega} T & 1 \\ -T_2 D_{\sin}^{\Omega} & S \mathbf{V} \begin{pmatrix} T_1 D_{\sin}^{\Omega} T_2^{-1} \\ T_2 D_{\cos}^{\Omega} T_2^{-1} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix} = R_1^T \Delta^{sq} R_2 \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{p}(0) \end{pmatrix}$$

if vacuum

experimental control

experimentally implementable squeezing (blue) and theoretical values (red)



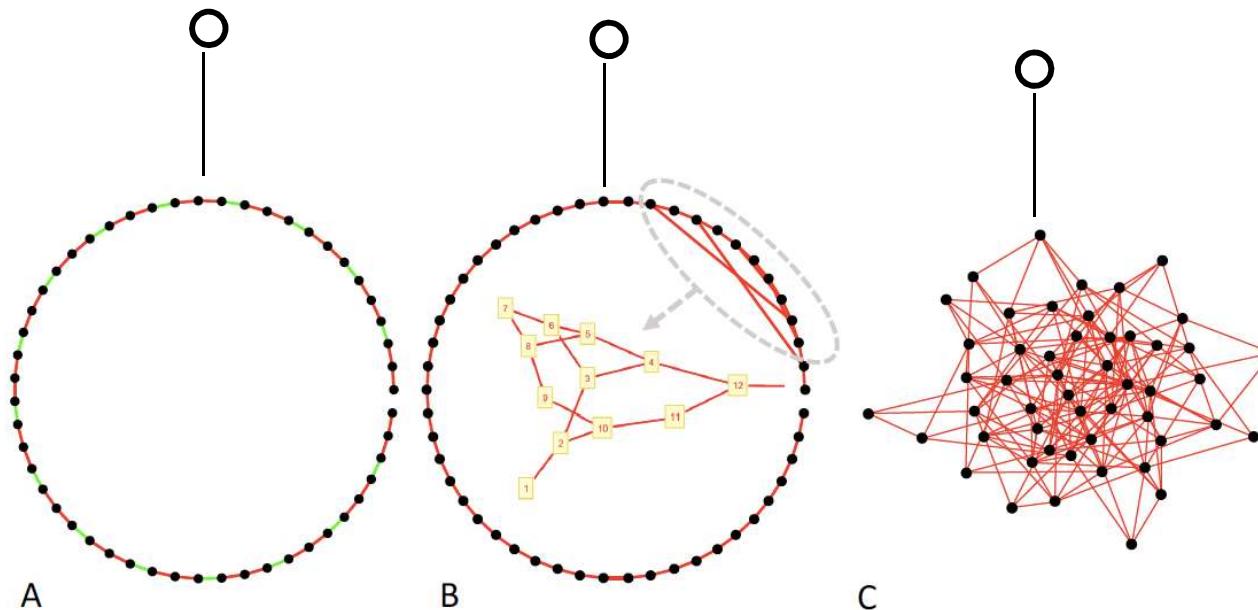
Ex: complex network = structured environment  
 additional oscillator= probe/system

*Additional oscillator*

$$H_S = (p_S^2 + \omega_S^2 q_S^2)/2, \quad H_E = \frac{\mathbf{p}^T \mathbf{p}}{2} + \mathbf{q}^T \mathbf{A} \mathbf{q} \quad H_I = -k q_S q_i$$

*Evolution given by  $H_E + H_S + H_I$*  

Measure the state of the probe  
 and recover the structure





# Quantum complex networks

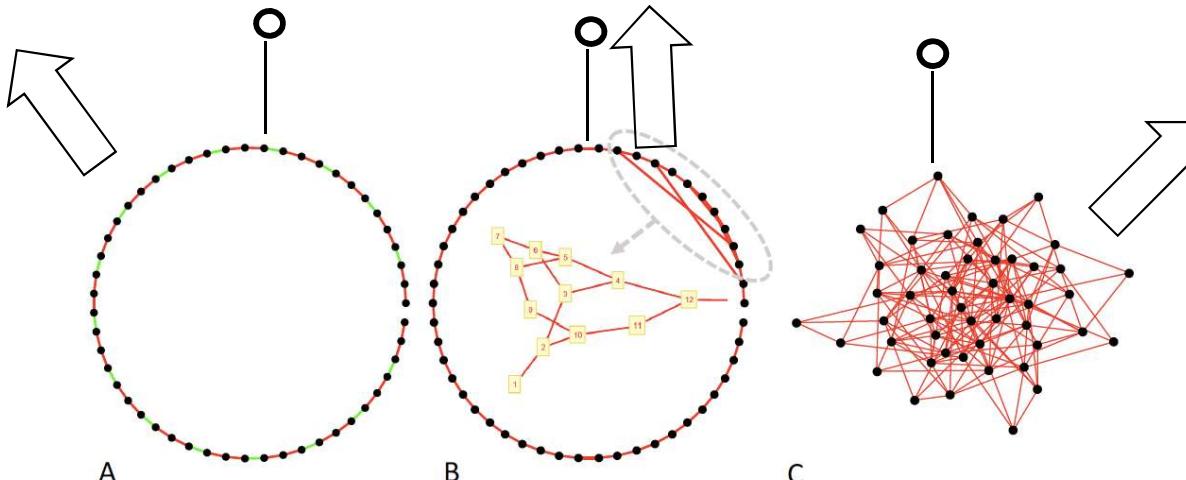
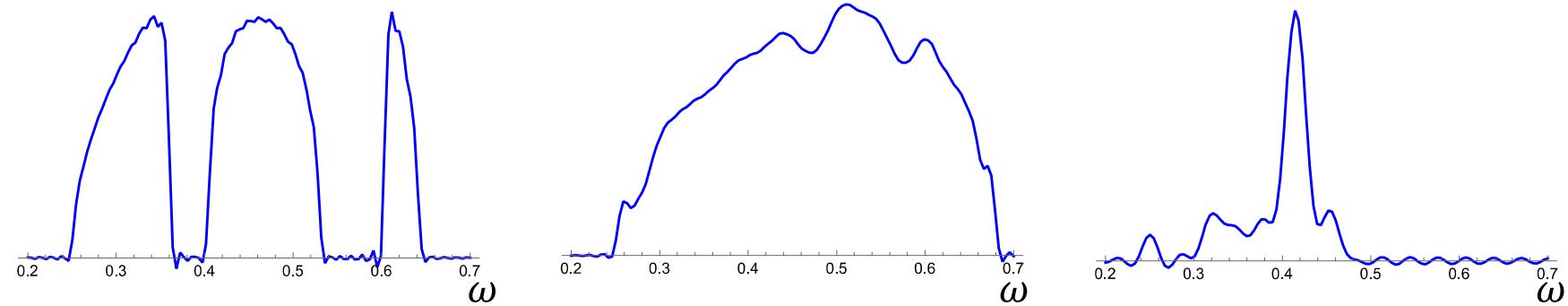
Ex: complex network = structured environment  
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$$J(\omega) = \frac{\pi}{2} \sum_i \frac{k^2 g_i^2}{\Omega_i} \delta(\omega - \Omega_i)$$

spectral density of environmental coupling

$$J(\omega) = \omega \int_0^{t_{max}} \gamma(t) \cos(\omega t) dt$$

Analytical



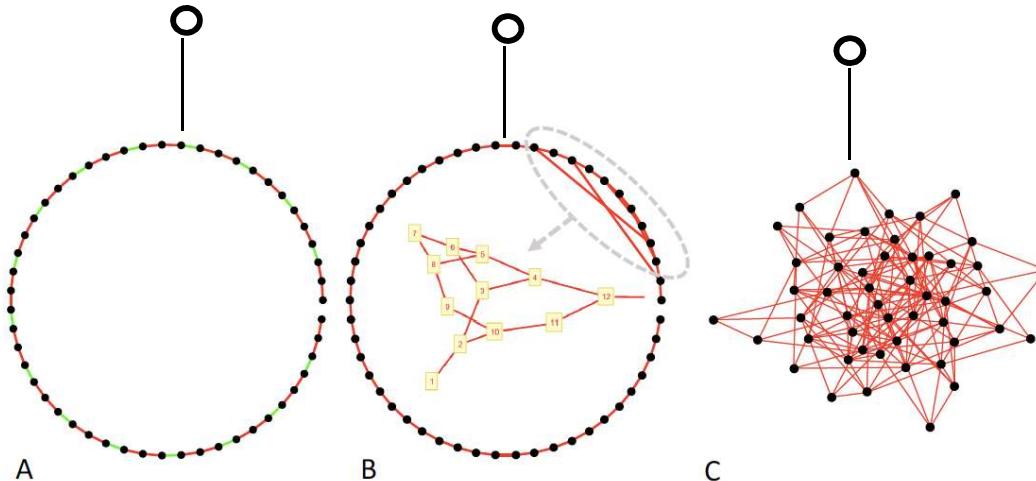
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$$\begin{pmatrix} \mathbf{Q}(t) \\ \mathbf{q}_S(t) \\ \mathbf{P}(t) \\ \mathbf{p}_S(t) \end{pmatrix} = \begin{pmatrix} O_1 D_{\cos} O_1^{-1} & O_1 D_{\sin} O_2^{-1} \\ -O_2 D_{\sin} O_1^{-1} & O_2 D_{\cos} O_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{Q}(0) \\ \mathbf{q}_S(0) \\ \mathbf{P}(0) \\ \mathbf{p}_S(0) \end{pmatrix}$$

Measure the state of the probe  
and recover the structure



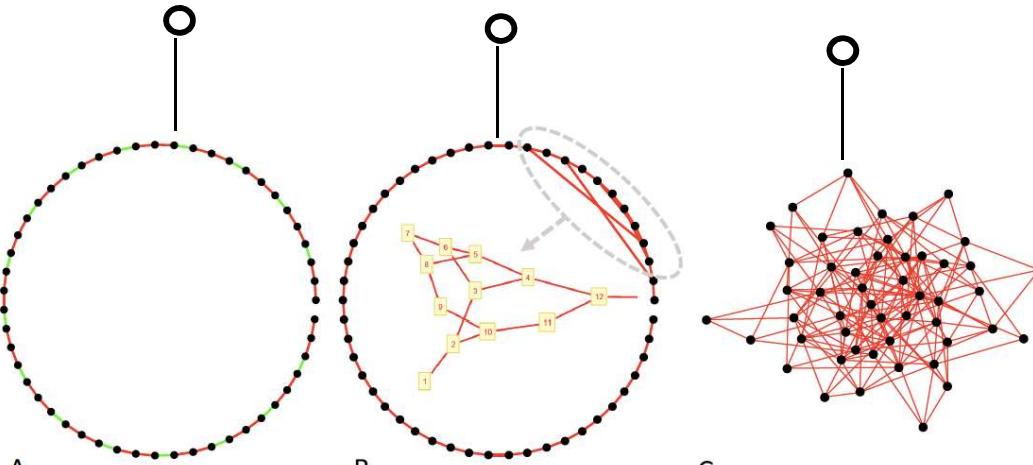
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Measure the state of the probe  
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Ex: complex network = structured environment  
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*Additional oscillator*

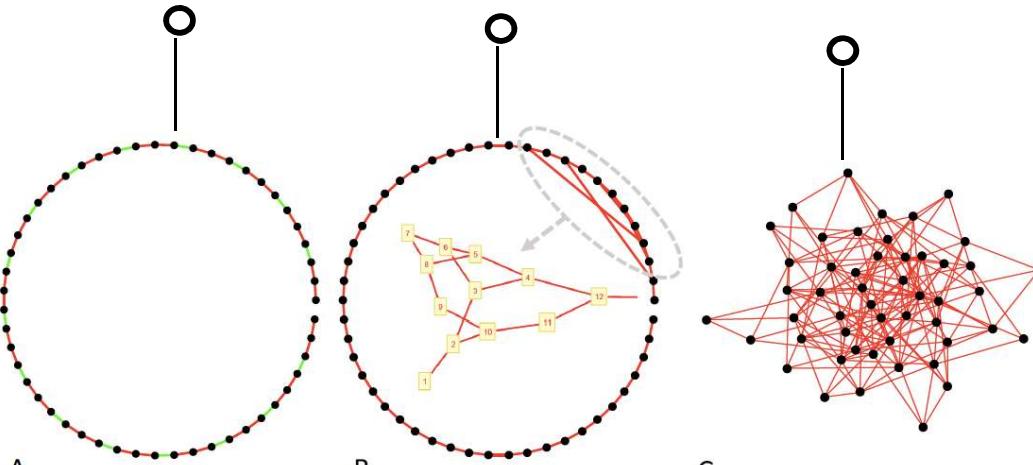
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Measure the state of the probe  
and recover the structure

*vacuum?*

*doesn't work*



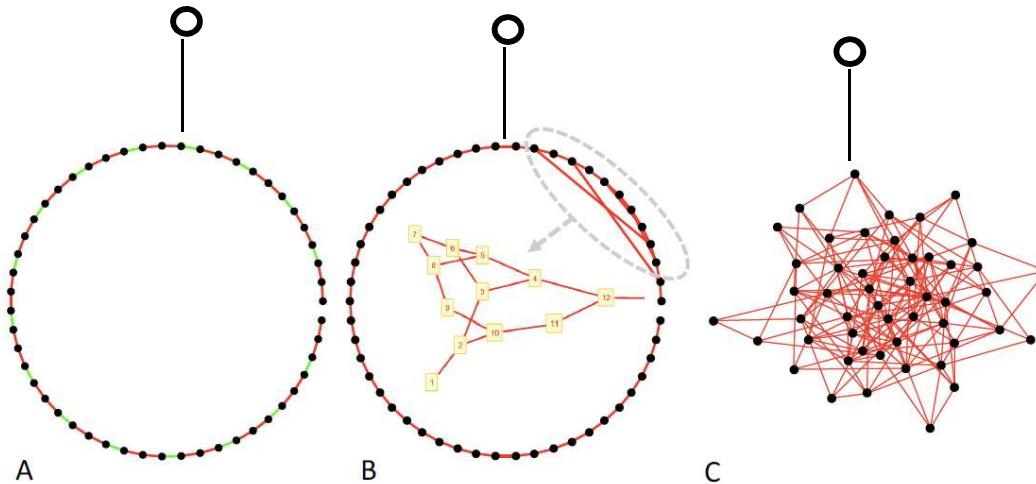
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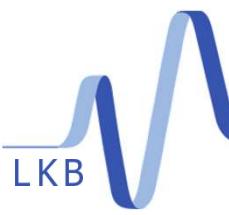
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*preparing at least  
the probe in a  
non-vacuum state*





# Quantum complex networks

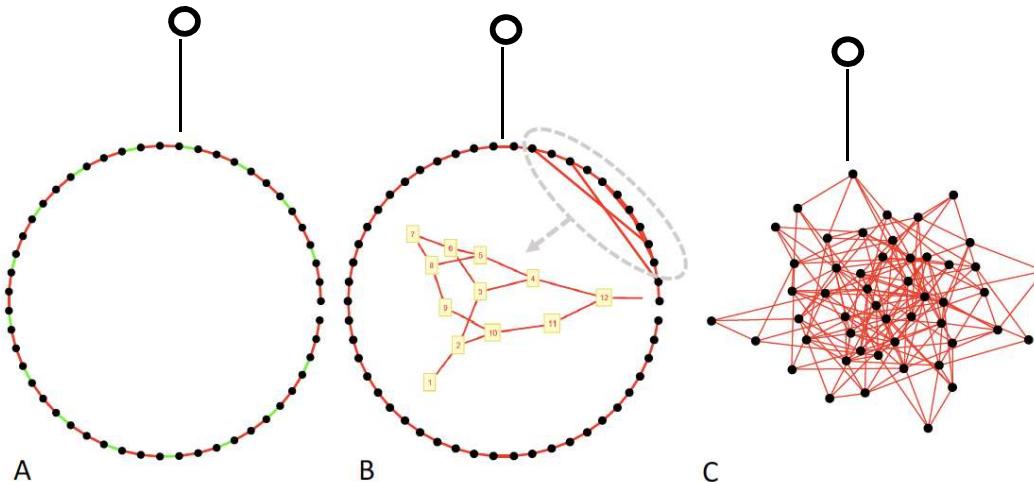
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*vacuum*



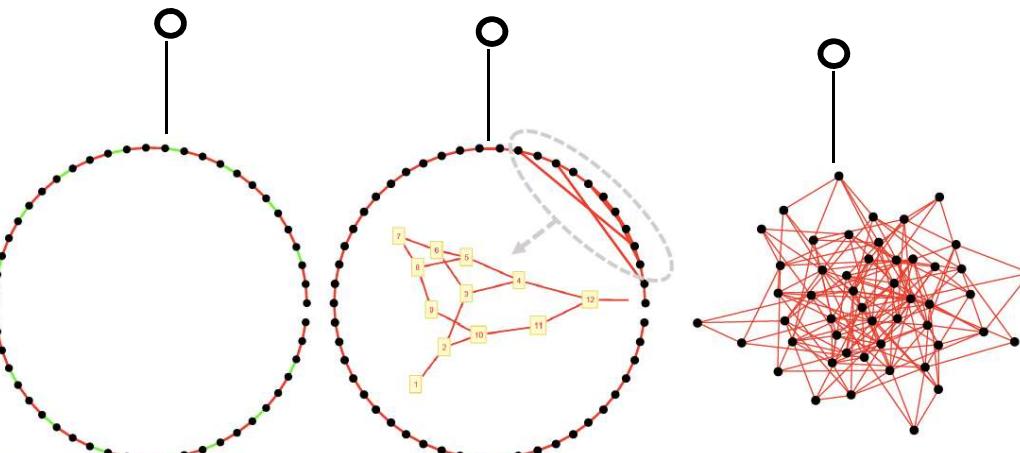
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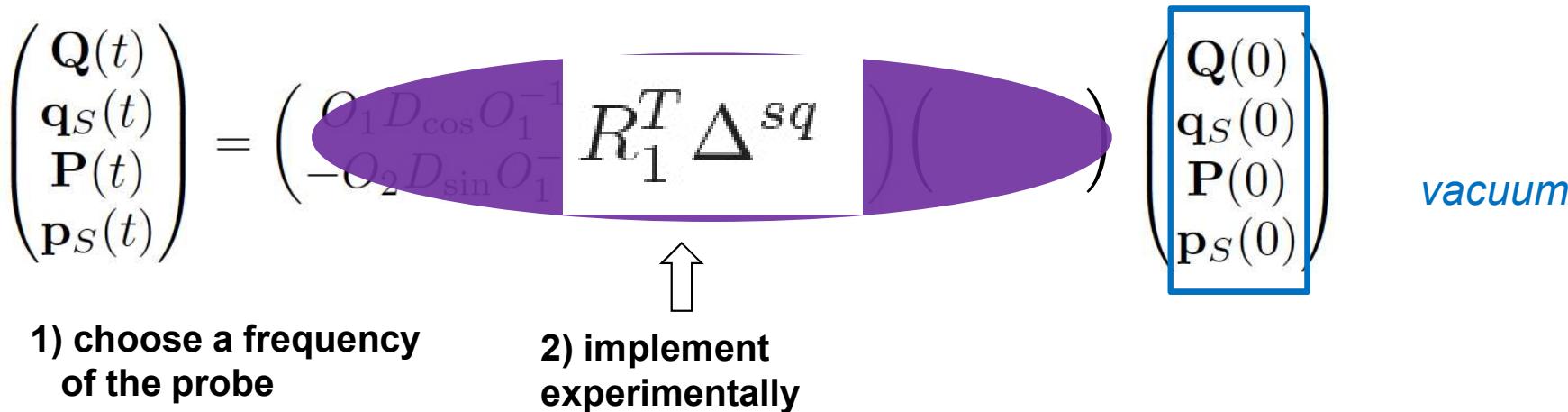
*vacuum*



Ex: complex network = structured environment  
 additional oscillator= probe/system

*Additional oscillator*

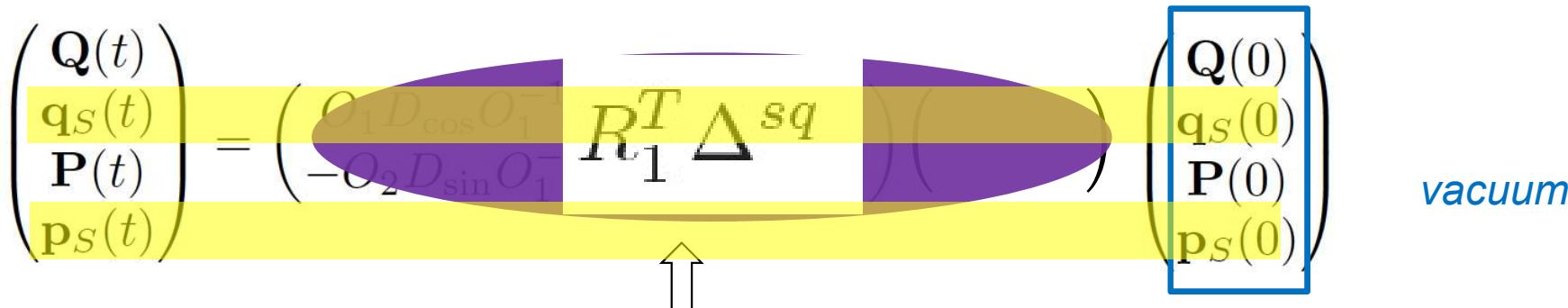
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Ex: complex network = structured environment  
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*Additional oscillator*

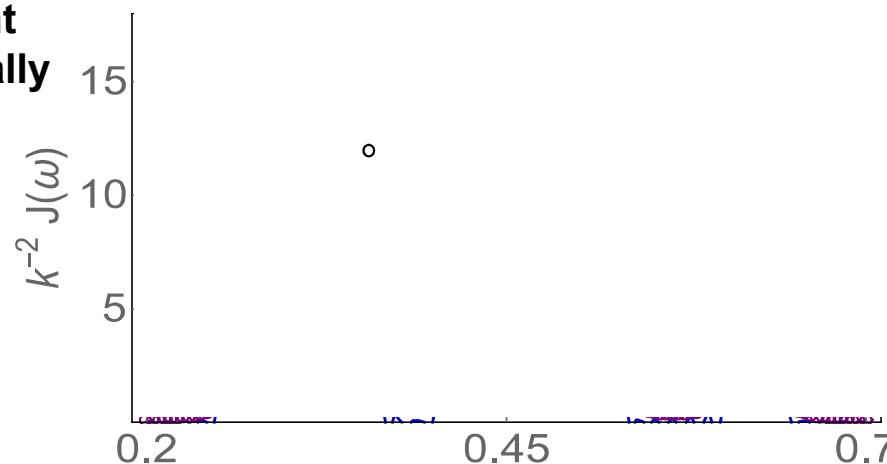
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1) choose a frequency  
 of the probe

2) implement  
 experimentally

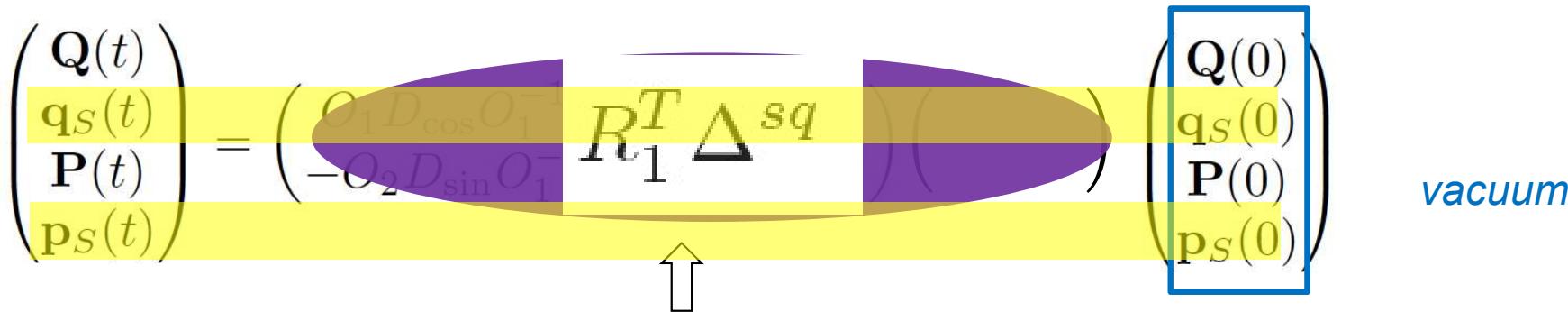
3) measure the probe  
 get a point



Ex: complex network = structured environment  
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*Additional oscillator*

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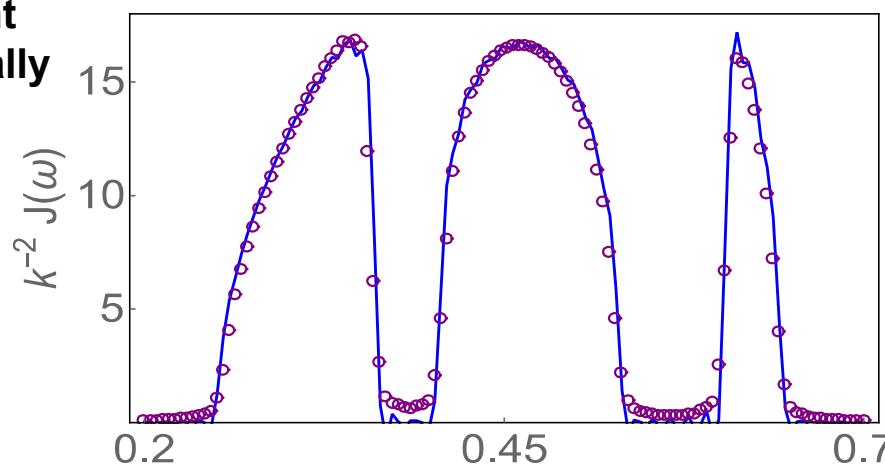


1) choose a frequency  
of the probe

2) implement  
experimentally

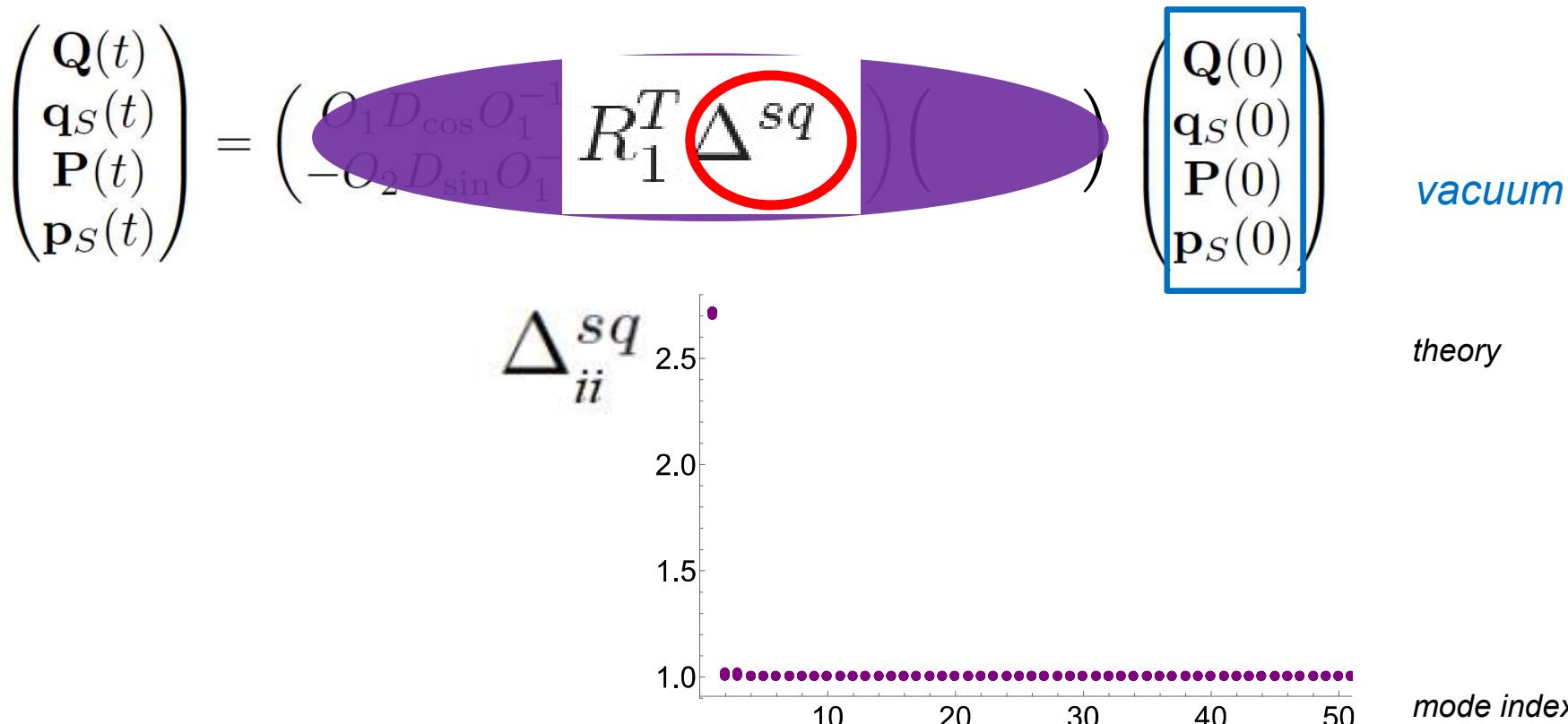
3) measure the probe  
get a point

repeat



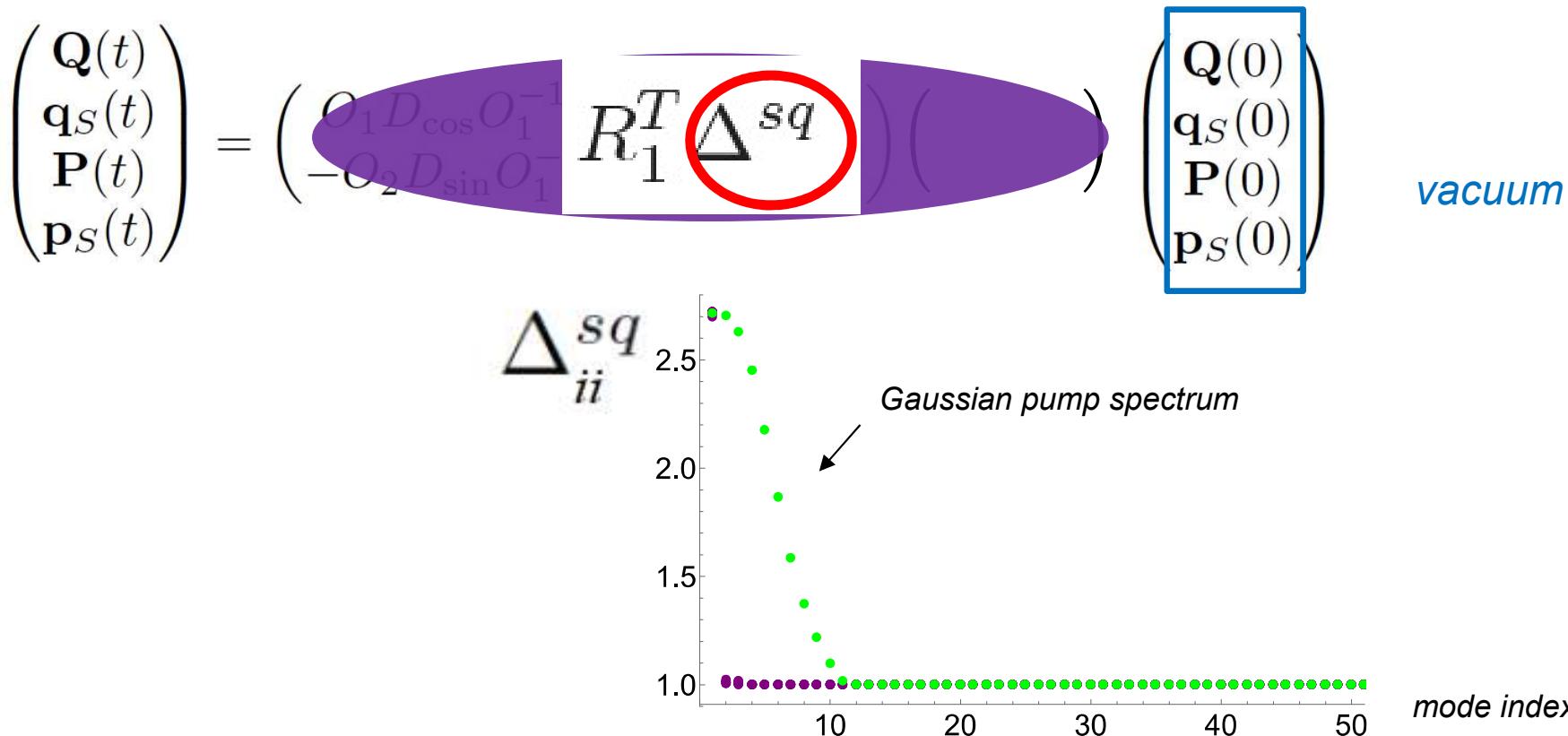
Ex: complex network = structured environment  
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## Experimental feasibility



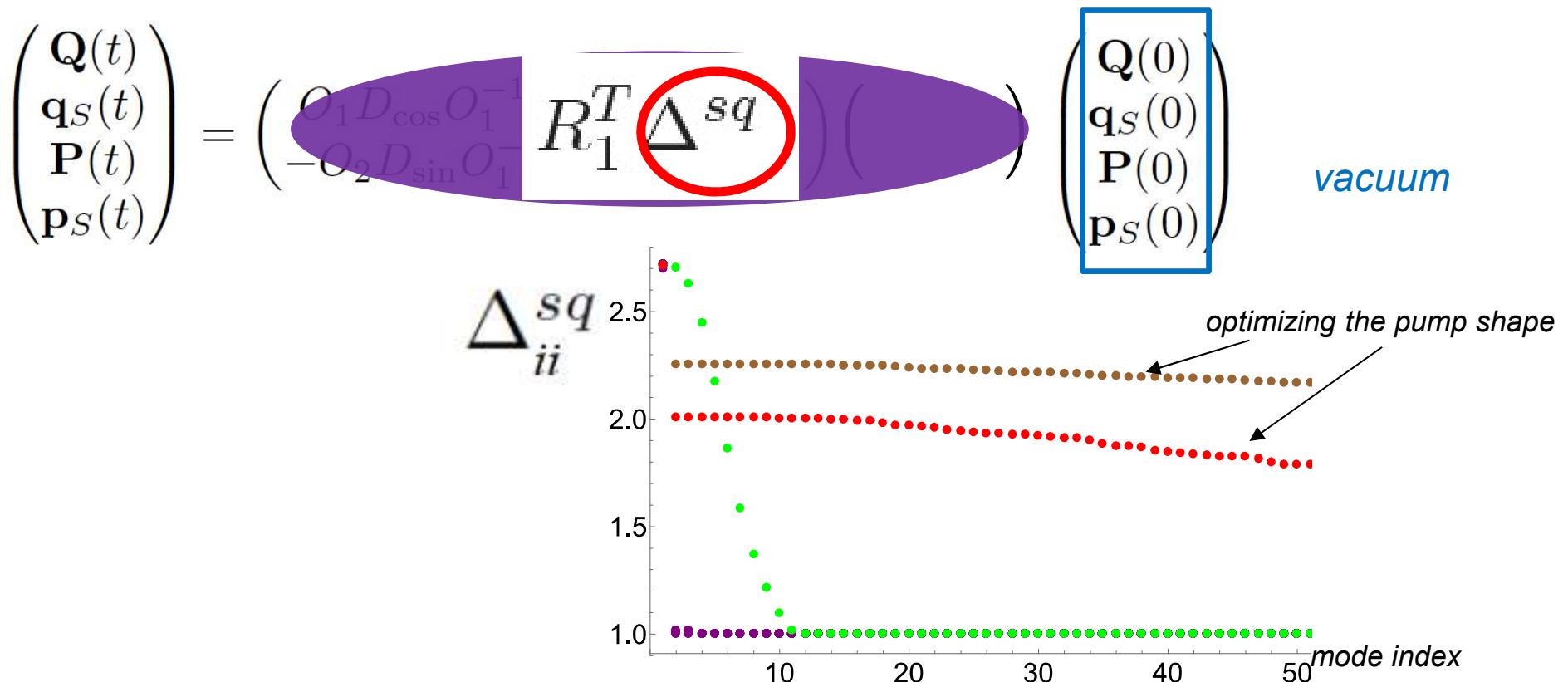
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## Experimental feasibility



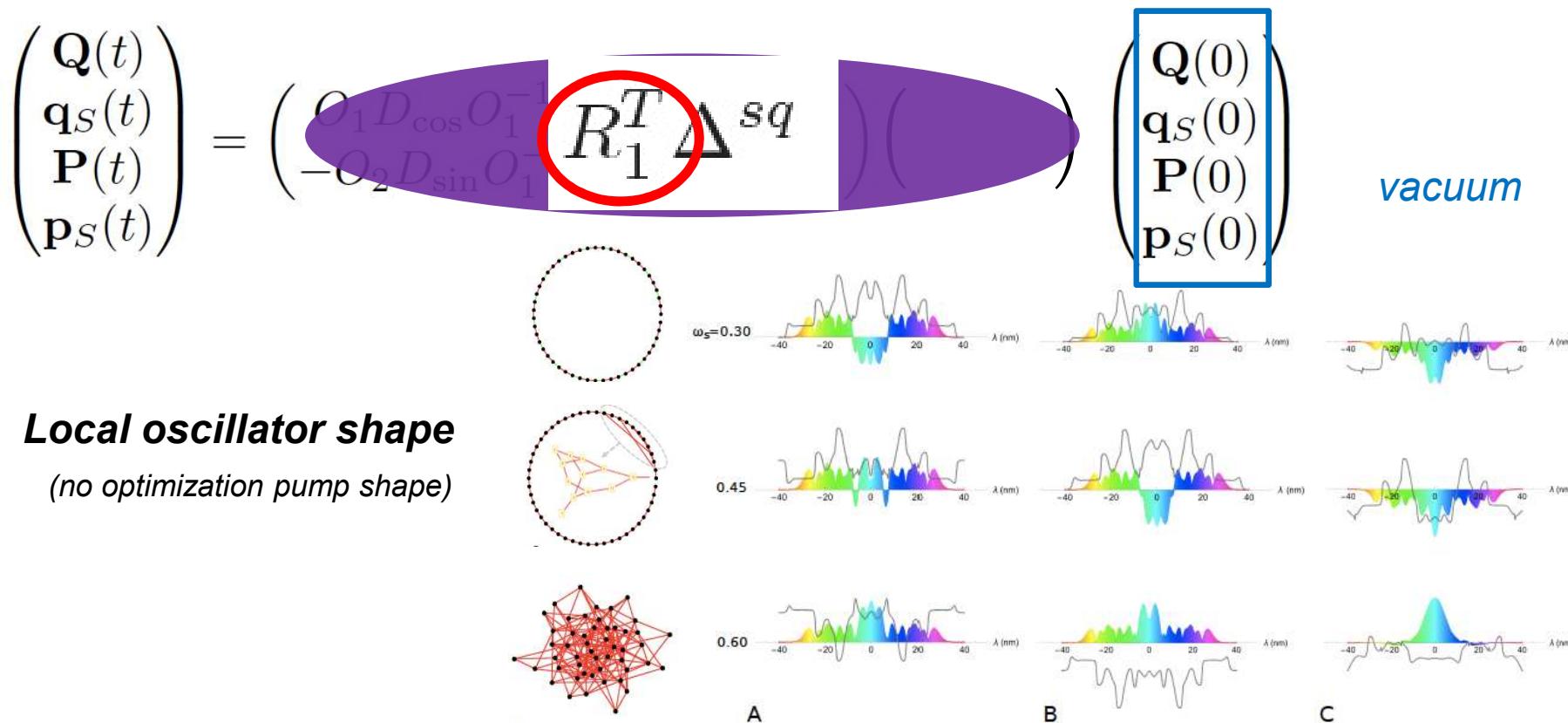
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## Experimental feasibility



Ex: complex network = structured environment  
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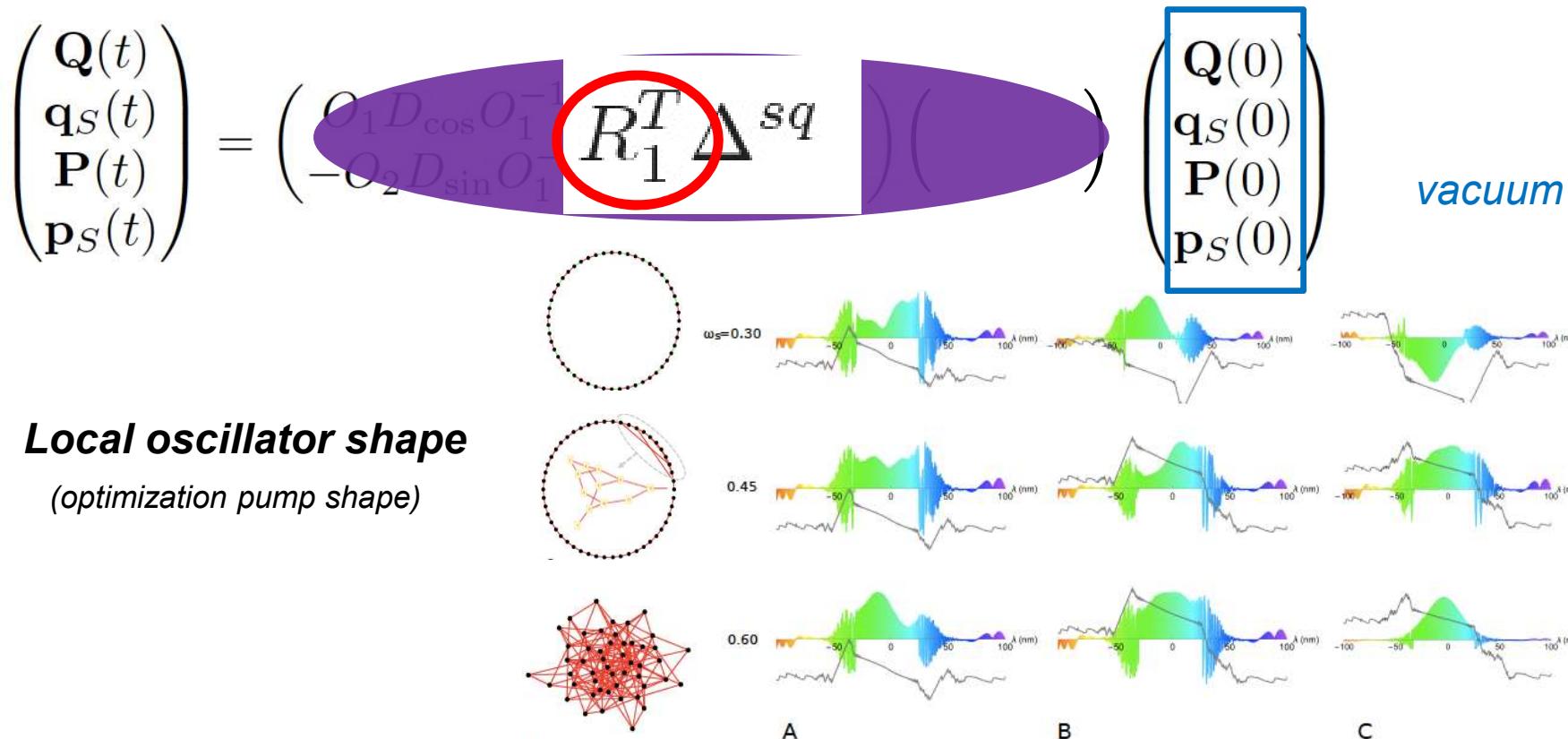


### Local oscillator shape

(no optimization pump shape)

Ex: complex network = structured environment  
 additional oscillator= probe/system

## Experimental feasibility

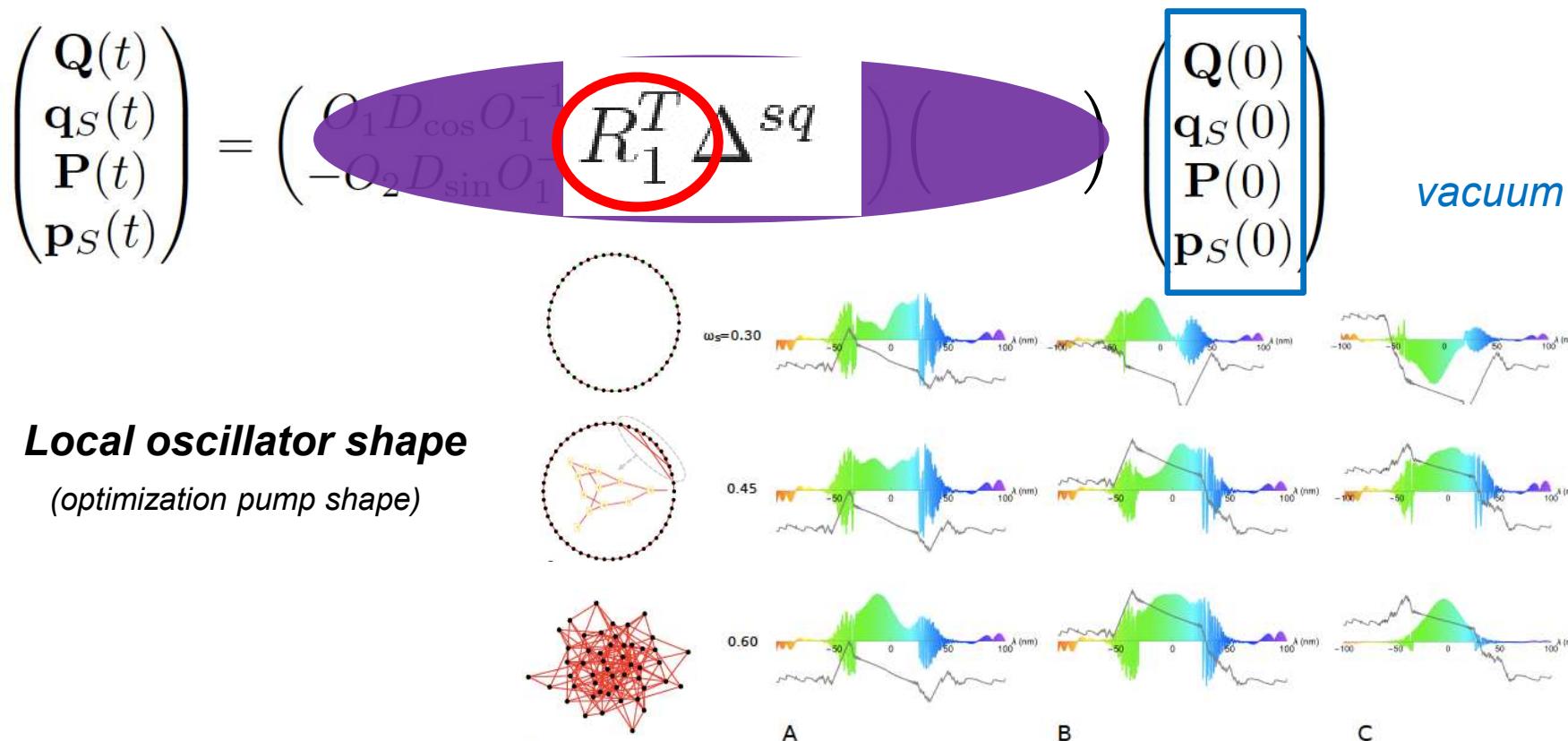


## Local oscillator shape

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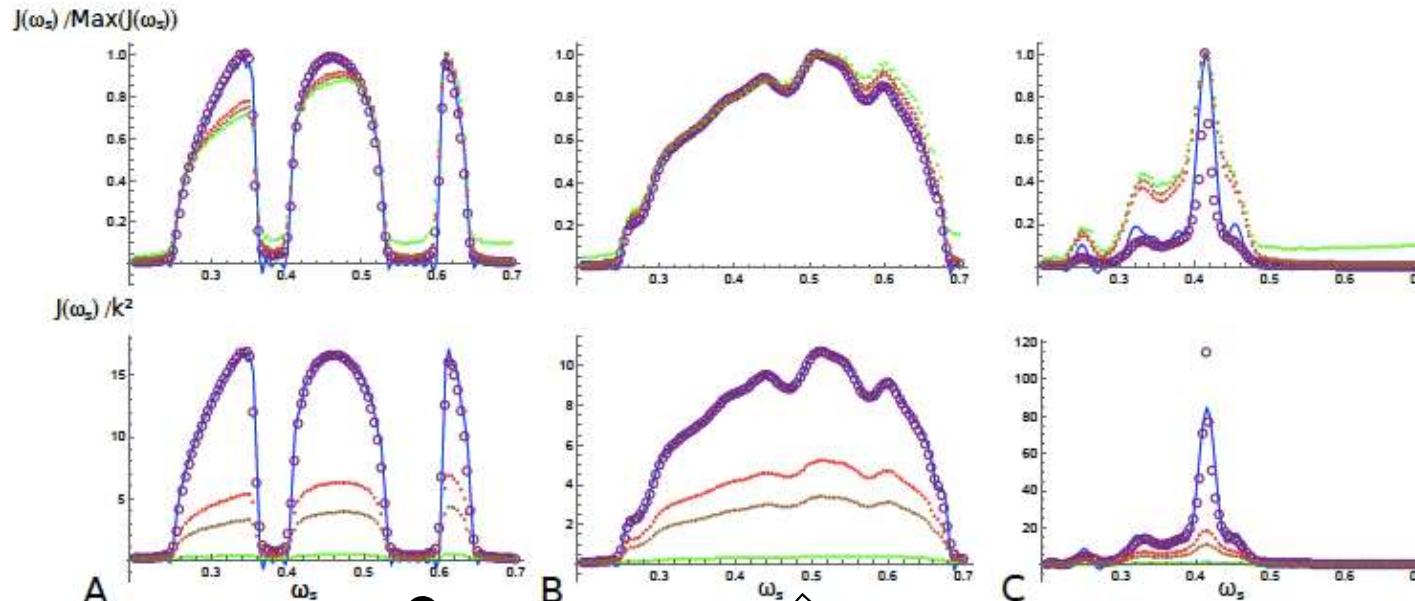


## Local oscillator shape

(optimization pump shape)

Ex: complex network = structured environment  
 additional oscillator= probe/system

## Experimental feasibility



## Results

